

MASSES OF QUARK CONFIGURATION

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ABSTRACT. *Within the bounds of the general relativity and in gravidynamics, spherically-symmetric configurations are considered with the limit equation of state ($P=(\varepsilon-4B)/3$) and with the density increasing to the center. It is shown that unlike GR, where the existence of strange stars only is permissible (u -, d -, s -quarks), in the consistent dynamic theory of gravitation the existence of stable configuration with $\varepsilon \propto r^{-2}$ (quark star) is possible with a "bag" out of quark-gluon plasma which includes all possible quark flavors (u, d, s, c, b, t, \dots). The total mass of such a compact object with the bag of the radius of ≥ 9 km (whose surface consists of the strange self-bound matter) must be $\geq 6 M_{\odot}$.*

1. INTRODUCTION

By the metric theory we mean here first of all general relativity (GR) and all versions of gravitational theories which proceed from Einstein's principle of equivalence. A project of theoretical model of gravitational interaction based on the consistent application of dynamic principles (gravidynamics) is presented more or less completely in the proceedings of XII and XIII workshops on high energy physics in Protvino (Sokolov, 1990, 1991; see also the references therein). In gravidynamics (GD) the law of equivalence of inertial and gravitational masses is certainly true, but here "the principle of equivalence" is not used in any way which we consider, following Fock (1961), only a kinematic consequence of the fundamental law: $m_i = m_j$. Accordingly, in the consistent dynamic theory of gravitation the field is not reduced to the space-time metrics, which as in Maxwell's electrodynamics, can always be described by Minkovsky's metric tensor.

In the suggested report we try to answer the question why in GR only quark confi-

gurations, consisting of strange matter (u, d, s quarks) - "strange stars" - are permitted, while in GD quark-gluon plasma of analogous objects may consist of quarks of all possible types - "quark stars" proper. As will be seen from the following, the peculiarities of GR and GD become most essential when we consider the utmost inhomogeneous quark configurations in both theories.

There are already many calculations of quark configurations (strange stars) within the bounds of GR, (i.e., the calculations on the basis of Oppenheimer-Volkoff's (OV) hydrostatics equations (Haensell et al., 1986, Alcock et al., 1986, Benvenuto and Horvath, 1989, Krivoruchenko, 1987, Overgard and Ostgaard, 1991). The same calculations were carried out recently in the SAO of the Russian AS. But unlike other groups we were interested first of all in the way the modern (dynamic) theory of strong interactions - quantum chromodynamics (QCD) - "works" in the conditions of the *strongest* gravitational field of a compact object with a mass of $\geq M_{\odot}$. Up to now the corresponding observational information still does not exclude alternatives to GR. Ultimately, our basic purpose is a *test* of the gravitational interaction theory. The elucidation of observational consequences and obtaining estimates allowing to compare GR and GD in a description of the same object - compact quark configuration.

2. UTMOST INHOMOGENEOUS QUARK CONFIGURATIONS IN GR

This section addresses the calculations of purely quark configurations described by the *limiting* equation of state

$$P_0 = \frac{1}{3} (\varepsilon - 4B), \quad (1)$$

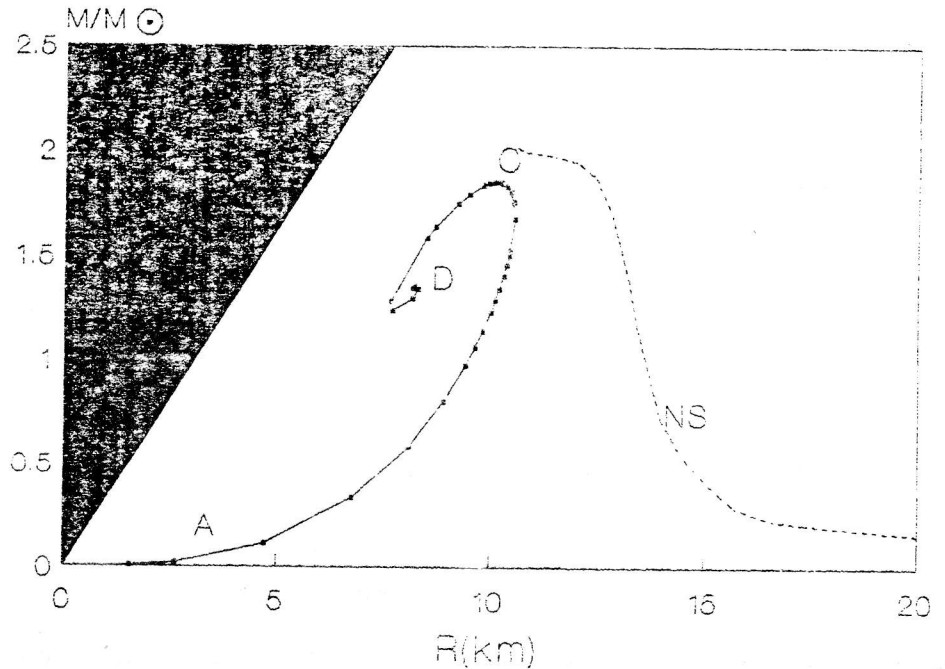
where ε is the total energy density inside a huge quark bag, $4B/c^2 = \rho_{\text{QGP}}$ is a macroscopic density on the surface ($P_0=0$) of the bag consisting of the quark-gluon plasma (QGP). Equation (1) is the limit to which tends the corresponding total equation of state (Alcock et al.; 1986, Haensell et al., 1986), describing the media consisting of quarks with masses tending to zero. Quark-gluon interactions stay in the lowest order in α_c (i.e., $2\alpha_c/\pi$ should be sufficiently small so that it remains still the first term of expansion in the expression for thermodynamic potential). Below, we speak about sufficiently cold (catalyzed) quark matter at temperatures not more than (for example) 10^{10} K when this matter is already a degenerate Fermi fluid. Thus in this paper the question is on a totally cooling down quark star when electrons are absent in the picture, and with no electrons, there is no corresponding neutrino flux. As a matter of fact, subject to the above remarks, we shall use here an asymptotic MIT bag model (Overgard and Ostgaard, 1991), where the bag constant B is a measure of confinement strength.

Integration of equations of hydrostatic equilibrium (OV equations) gives, in particular, the relation between the mass and the radius of the compact object which

is shown in Fig.1. Here the value B is chosen as $B=67 \text{ MeV/fm}^3$, that corresponds to the macroscopic density on the surface ($P_0=0$) of QGP-bag equal to $\rho_{\text{QGP}} \approx 1.7\rho_{\text{nucl}}$ ($\rho_{\text{nucl}} = 2.8 \cdot 10^{14} \text{ g/cm}^3$).

Usually all the calculations of this kind (see, for example, Haensell et al., 1986; Alcock et al., 1986) are interrupted near point C in Fig.1., that corresponds to the OV limit for a compact object with the equation of state (1). All configurations lying to the left of point C turn out to be unstable with respect to small radial perturbations, branch $C-D$ in Fig.1 is closest to the black holes - "permanently" unstable objects. Such an instability is described in many manuals on GR (Zel'dovich and Novikov, 1971); Shapiro and Teukolsky, 1983; Garrison et al., 1965).

Fig.1. The mass-radius relation for purely quark configurations with equation of state (1). The dotted line shows the M/R relation for neutron stars. Point C corresponds to OV-limit, point D indicates the position of the utmost inhomogeneous quark configuration in GR. The black hole region is shaded (see the text). $B=67 \text{ MeV/fm}^3$.



In Fig.1 the dotted line shows the M/R connection for neutron stars. This connection is close to that given by Bete-Johnson's equation of state. The same connection M/R at $B = 67 \text{ MeV/fm}^3$ corresponds at first (i.e., for small masses $\leq 0.5 M_{\odot}$) to neutron stars, and then to neutron stars with a growing (with further mass increase) quark nucleus, arising inside the compact object when its central density exceeds the value $4B/c^2$ (see, in detail, Haensell et al., 1986).

From Fig.1, it is seen that at the same mass, neutron stars are more extended, or less compact objects, than purely quark configurations calculated for limit equation of state (1). In GR only black holes with infinite gravitational red shift z can be more compact objects. Thus, if equation (1) is really the limit of the equation of quark matter state at superhigh densities ($> \rho_{\text{nucl}}$), then it means that, in the bounds of GR, simply, there are no more compact hydrostatic equilibrium configurations than those shown in Fig.1. In this sense, purely quark configurations corresponding to the curve AC in Fig.1 are the limit objects for GR. Here we can speak of hydrostatic equilibrium stable compact objects with the surface (z is a finite value)

instead of the event horizon.

For such object (with the surface), limit in the GR, we can say about limit maximum value of the gravity acceleration (strength of the gravitation field). The dependence between the mass of a quark star and gravity acceleration on the surface for this object is shown in Fig.2. As is shown in Fig.2, when the mass of a quark star reaches maximum, then gravity acceleration on the surface reaches some critical value too. The matter of the star cannot resist the action of gravity and the star is getting unstable relative to collapse.

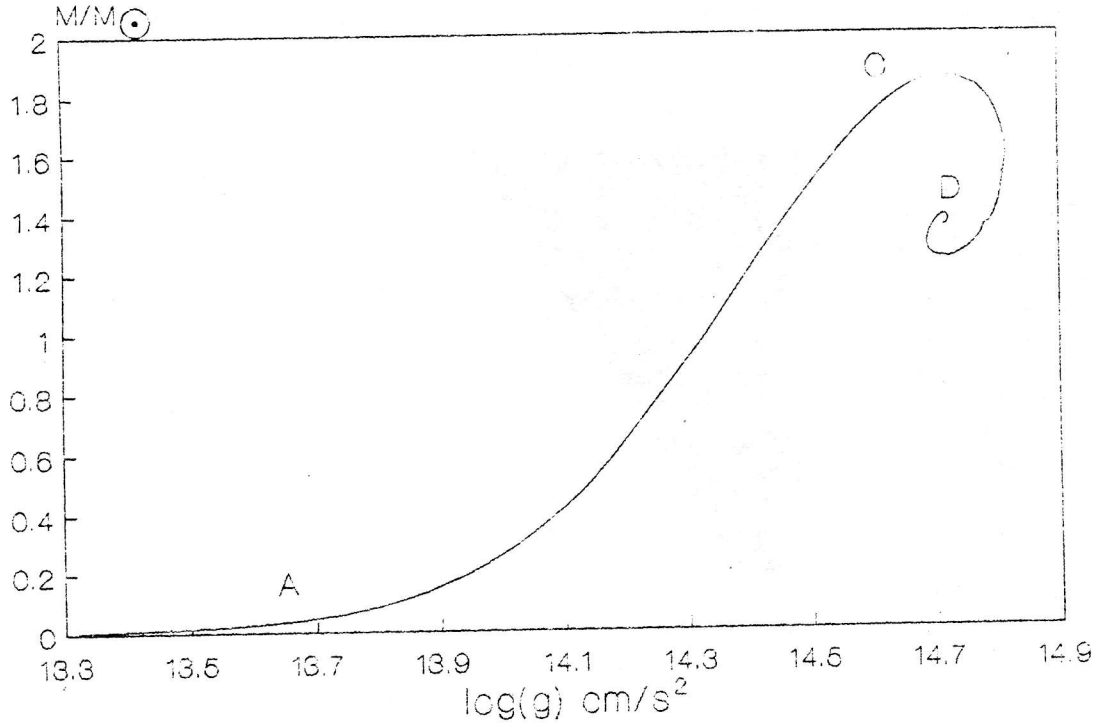


Fig.2. The mass-gravity acceleration for purely quark configuration with equation of state (1). Point C corresponds to OV limit, point D indicates the position of the utmost inhomogeneous quark configuration in GR. $B=67 \text{ MeV/fm}^3$.

But the basic difference between purely quark configurations and neutron stars which is important for us to underline here, is the fact (and it is seen in Fig.1) that the quark matter or strange matter in GR is *self-connected*. Such an object is really a huge quark bag whose hydrostatics at small masses (A branch in Fig.1) is guaranteed only by strong (colour) interaction. At $M > M_\odot$ and at masses close to the OV limit the curve in Fig.1 "turns" to black holes due to GR effects.

Hydrostatic calculations with the use of OV equations give the following dependence on the value of B for the maximum mass of purely quark configurations:

$$M_{\max}(C) \approx 1.85 M_\odot \left(\frac{67 \text{ MeV/fm}^3}{B} \right)^{1/2} \quad (2)$$

The analogous equation for the OV limit was obtained by Haensel et al. (1986) as a result of numerical calculations including the equation of state (1). From this paper

and also from the calculation by Alcock et al. (1986) it follows that the basic parameter which determines the value of masses and radii of such dense ($>\rho_{\text{nuc1}}$) and compact objects - quark configurations - is the value B . Our calculations have confirmed the conclusion that the crucial factor is the choice of the macroscopic density value $\rho_{\text{QGP}} > \rho_{\text{nuc1}}$ at the surface of the QGP-bag.

In other words, equation of state (1), in which quarks are considered in the limit as almost free noninteracting massless particles at the calculation of the hydrostatics of objects with such high densities, gives approximately the same values of the "observed" parameters as the equation of state does, allowing for a finite value of the constant of colour interaction and nonzero mass of s -quark. As it will be seen from the following, this circumstance can be directly interpreted in the bounds of consequences of QCD and macroscopic properties of QGP.

The choice of $4B/c^2$ density on the surface ($P_Q=0$) of the macroscopic quark bag can be determined from the following reasons.

The upper value of the constant B is determined by the condition of the self-connection of quark (strange) matter at zero pressure P , formulated by Witten (1984). This condition demands that the corresponding energy per baryon at $P=0$ should be less than energy per baryon for the most stable non-strange matter - crystal iron:

$$E_Q(P=0) = \frac{4B}{n_s} = 860.6 \text{ MeV} \cdot \left(\frac{B}{67 \text{ MeV/fm}^3} \right)^{1/4} < 930.4 \text{ MeV} (^{56}\text{Fe}). \quad (3)$$

Hence, for the B constant, we obtain that $B \leq 91.5 \text{ MeV/fm}^3$, or for macroscopic density of plasma on the surface of the QGP-bag we obtain $\rho_{\text{QGP}} \leq 2.3\rho_{\text{nuc1}}$. It means that at a big leap of density on the bag surface some hadron configuration would be energetically more preferable than QGP.

The lower value of B is determined by the fact that with decreasing dimensions and mass of the bag (A branch in Fig.1) we shall come ultimately to a model of bags (MIT) or to a "macroscopic" configuration with baryon number $A \approx 100$ (consisting mostly of u - and d -quarks?). Thus, the masses of macroscopic quark configurations must be connected with hadrons mass spectra by the (semi-empiric) relation (Chodos et al., 1974)

$$B \geq B_{\text{MIT}} = 0.13 |\varepsilon_v| \approx 67 \text{ MeV/fm}^3. \quad (4)$$

Here $|\varepsilon_v|$ is the energy density of QCD-vacuum equal to $\approx 0.5 \text{ GeV/fm}^3$ (Novikov et al., 1981).

Thus the density at the surface of the macroscopic quark bag is somewhere within the limits

$$2.3 \rho_{\text{nuc1}} \geq \rho_{\text{QGP}}(P=0) \geq 1.7 \rho_{\text{nuc1}}. \quad (5)$$

Accordingly, maximum mass of purely quark configuration must be in the limits $1.58 M_{\odot} \leq M_{\text{max}}(C) < 1.85 M_{\odot}$. This is another consequence of the calculations of the

kind (Haensel et al., 1986; Overgard and Ostgaard, 1991). Namely, the application of the modern phenomenology of strong interactions reduces considerably the value of the OV-limit. In the old phenomenology with Yukawa's potential and the exchange of vector mesons, where the equation of the type of $P=\varepsilon$ is used as a limit equation of state at $\varepsilon/c^2 \geq \rho_{\text{nuc1}}$, the corresponding value of the OV-limit is more than $3 M_{\odot}$ (Rhoades and Ruffini, 1974; Shapiro and Teukolsky, 1983).

To understand why, in GR, only strange stars are possible it is necessary to apply to the profiles of (energy) density the corresponding hydrostatically balanced quark configurations. In Fig.2 the behaviour of (energy) density is shown inside the quark bag as dependent on the distance from the center of a spherically-symmetric configuration. This density profile corresponds to the OV-limit, i.e., this is $\varepsilon(r)/c^2$ for the last stable hydrostatically balanced configuration which can exist in nature, if the equation of state (1) for QGP is true and, of course, if GR is true. Thus, GR together with limit equation of state (1), imposes the limitation on the maximum achievable density QGP (density in the center):

$$\rho \leq \rho_{\text{GR}} \approx 10 \rho_{\text{nuc1}} \approx 28 \cdot 10^{14} \text{ g/cm}^3 \quad (6)$$

and, consequently, on numerical value of the OV-limit.

All other stable configurations on the basis of Equation (1), lying to the right of point C, are even more homogeneous. In the limit, as was mentioned above, at $r \rightarrow 0$ (branch A in Fig.1) we deal ultimately with the model of MIT-bag with an absolutely homogeneous profile $\varepsilon(r)$ at baryon number $A \approx 100$. We emphasize here once more that purely quark configurations, corresponding to points on the curve from A to B in Fig.1, are the most homogeneous ones from possible stable compact configurations corresponding to OV-equations (Alcock et al., 1986).

After all, we can calculate the profile $\varepsilon(r)/c^2$ of the utmost inhomogeneous hydrostatically balanced configuration corresponding to point D in Fig.1. This profile is shown in Fig.3. The density in the center of such an object tends to infinity and falls, with r increase, very close to the law:

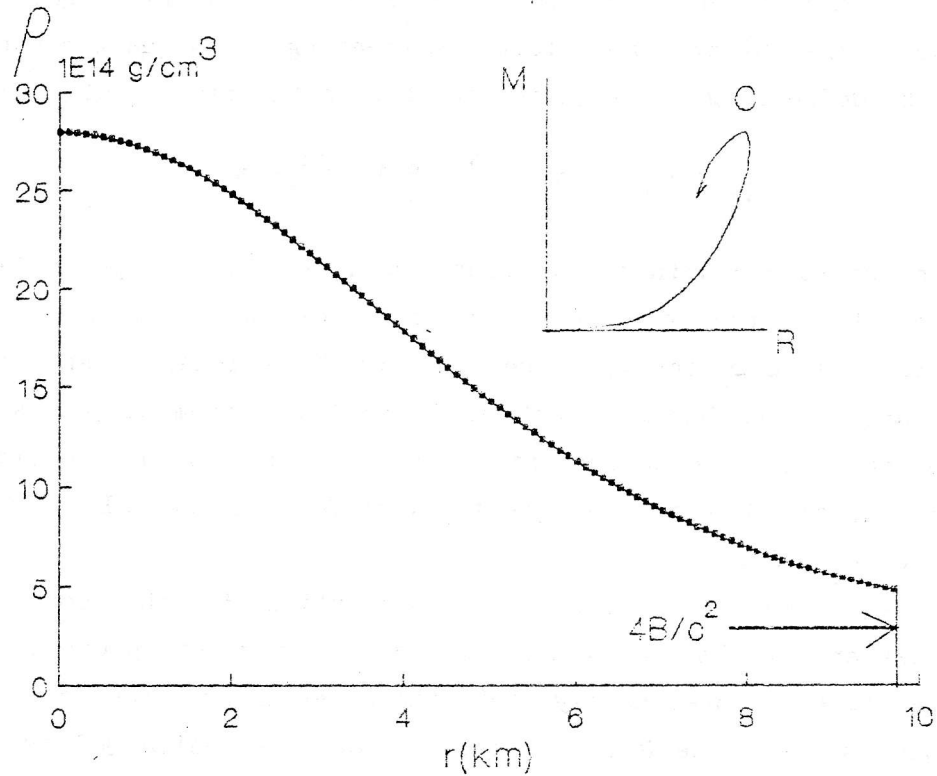
$$\varepsilon(r) \propto r^{-2}.$$

But such configurations, according to GR, are never realized in nature since they are most unstable with respect to small radial perturbations (Zel'dovich and Novikov, 1971) and during the time of the order of R/c they must collapse into black holes. But nevertheless we consider the situation in more detail, since it is such a configuration (forbidden in GR with big "margin") which can be realized as a stable stationary state in the bounds of the dynamic alternative to GR - in gravodynamics.

For macroscopic density at the bag surface, in accordance with conditions (5) we choose for definiteness some average value

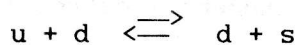
$$\rho_{\text{QGP}}(P_0=0) = 2\rho_{\text{nucl}} \approx 5.6 \cdot 10^{14} \text{ g/cm}^3. \quad (7)$$

Fig.3. (Energy) density profile for the last stable (OV-limit) spherically-symmetric configuration corresponding to point C in Fig.1. The distance r from the center of QGP-bag is measured in km, total density in g/cm^3 , $B=67 \text{ MeV/fm}^3$.



In accordance with QCD (the theory of colour interactions) and also in accordance with what is known about expected properties of QGP (Emel'yanov et al., 1990; Collins and Perry, 1975), it can be considered that the bag surface consists mainly of the lightest u - and d -quarks which come first in the stage of deconfinement at such a gigantic macroscopic density. (Approximately at the same density (7) the phase transition to QGP state occurs (see Haensel et al., 1986)). It can be interpreted so (analogously to usual plasma) that the free path length l (relative to colour interactions) of u - and d -quarks in such a plasma with $\rho \gg \rho_{\text{QGP}}$ becomes either equal or even much greater than $l_c \approx 1 \text{ fm}$; l_c being the characteristic radius of strong interaction or the radius of confinement. In the end, l can simply become a macroscopic value comparable with the bag dimension. These are just the quarks for which at $\rho \approx \rho_{\text{QGP}}$ the equation of gas state of asymptotically free quarks and gluons (1) turns out to be true, since for them we can assign $m_m \approx m_d \rightarrow 0$ (at such characteristic transmissions of momentum in QGP which correspond to particle interactions in such dense matter).

A heavier s -quark ($m_s \approx 200 \text{ MeV}$) at $\rho \gg \rho_{\text{QGP}}$ exists in plasma as heavy "admixture" as a result of equilibrium reactions of the type



(see the review by Haensel, 1987). Color interaction of this ("more non-relativistic", than u and d) quark is still rather strong ($\alpha_c > 0.45$) at $\rho \approx \rho_{\text{QGP}}$ and that is why

s-quarks must have *smaller* mean-free-path in QGP than "massless" u- and d-quarks.

So one can say that a heavier s-quark arising at weak interactions at $\rho \approx \rho_{\text{QGP}}$ is still mainly in the volume of confinement $l_c^3 \approx 1 \text{ fm}^3$. Density boundary over which the deconfinement or almost total "defreezing" of s-quark occurs is rather close to ρ_{QGP} . "The defreezing" of s-quark occurs when the macroscopic density becomes greater than

$$\rho_s \equiv \rho_{\text{QGP}} + m_s c^2 / l_c^3 \approx 9.1 \cdot 10^{14} \text{ g/cm}^3, \quad (8)$$

in accordance with the interpretation of chemical potential as the change of energy density at the unit change of concentration of particles of a given kind. In other words, at $\rho > \rho_s$ in every "cell" of $\approx 1 \text{ fm}^3$ volume, there is already more than one s-quark and it becomes just as "tight" for them as it was at $\rho > \rho_{\text{QGP}}$ for u- and d-quarks. Then the colour interaction of s-quarks must become weaker ($\alpha_c < 0.45$). Accordingly, at QGP densities greater than ρ_s , s-quarks also can be considered relativistic ($m_s \rightarrow 0$).

As a matter of fact, the confirmation of such a logic of "defreezing" of s- (and heavier) quarks is the fact noted in the quoted paper by Haensel et al. (1986) and mentioned before, namely, that the determining parameter of the equation of state for QGP is the value B or the density value at which QGP is formed. The direct calculation of the hydrostatically balanced configuration with the limit equation of state (1), not only reproduces the results of that paper, where the authors, besides the B , allow also for $\alpha_c = 0.45$ and $m_s c^2 = 200 \text{ MeV}$, but varying $4B/c^2$ value in (1), one can reproduce, with an acceptable precision, almost all the results of calculations by Benvenuto and Horvath (1989) with other parameters α_c and m_s . Hence, we conclude that indeed the densities ρ_{QGP} and ρ_s (at which s-quark can also be considered free massless particle) must be close $\rho_{\text{QGP}} \sim \rho_s$. I.e., Equation (1) can actually be applied without paying attention to the essential difference in masses between u-, d- and s-quarks.

Or, in other words, the following *interpretation* of the results of all the mentioned calculations is possible. At $\rho_{\text{QGP}} \leq \rho \leq \rho_s$, the heavier quark is present in QGP as "a heavy admixture" and does not distort strongly the limit equation (1) and at $\rho > \rho_s$ heavy quarks, as well as lighter ones, are also in the state of deconfinement and so here equation (1) turns applicable again.

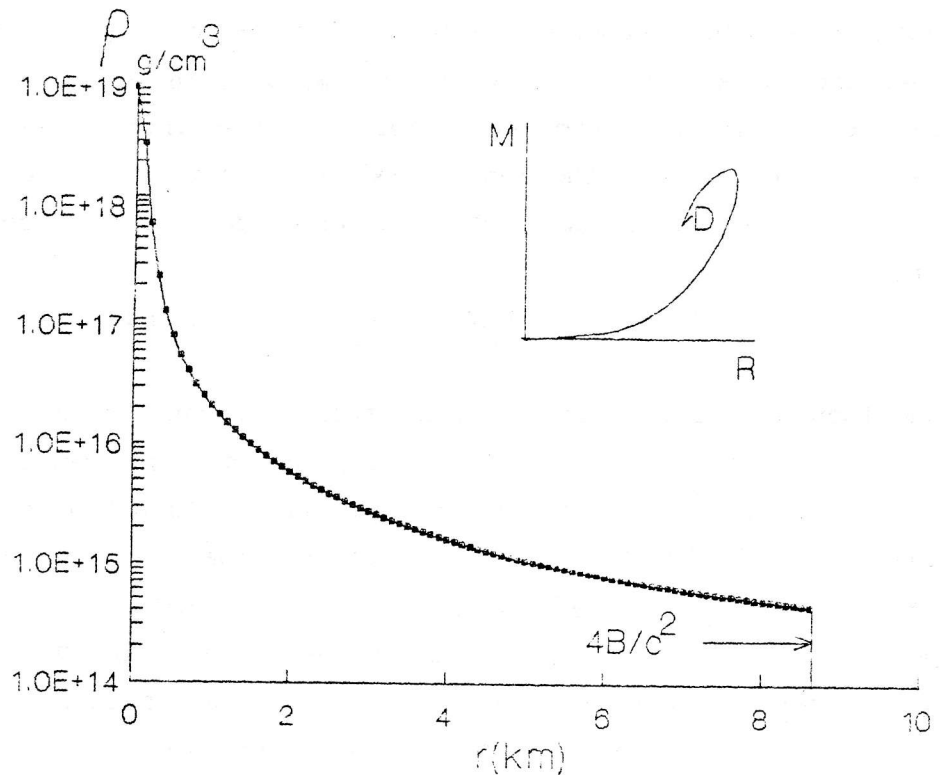
Now from what is said above, it becomes clear why in GR only strange stars are possible. As has been noticed the last stable hydrostatically-balanced configuration (see Fig.3) imposes limitation on density (6). Only then one can speak about strange stars following the same logic, the deconfinement ("defreezing") of even heavier c-quark ($m_c c^2 = 1.4 \text{ GeV}$) must occur at densities greater than the limit density ρ_{GR} . In every "cell" of l_c^3 volume there can be at least one c-quark if the macroscopic density turns greater than

$$\rho_c \equiv \rho_s + m_c c^2 / l_c^3 \approx 33.9 \cdot 10^{14} \text{ g/cm}^3. \quad (9)$$

In that case the colour interaction becomes so weak that the corresponding α_c becomes even less. Therefore, one can consider for c -quark that $m_c \rightarrow 0$. Of course, for the configuration in Fig.3, which is still attainable in GR, some admixture of c -quarks can appear in plasma near the center as a result of some $(d+u \rightarrow d+c)$ weak processes analogously with the admixture of s -quarks at $\rho < \rho_s$...

Returning to the utmost inhomogeneous configuration with the density profile in Fig.4, at $\rho > \rho_c$ in plasma there must be a lot of relativistic charmed quarks ($m_c \rightarrow 0$) and like the case of s -quark the properties of QGP are described ultimately by limit equation (1).

Fig.4. (Energy) density profile for the utmost inhomogeneous hydrostatically-balanced configuration with equation of QGP state (1) calculated in the bounds of GR. The density on the QGP-bag is taken to be equal to $\rho_{\text{QGP}} = 1.7 \rho_{\text{nuc1}}$ ($B=67 \text{ MeV/fm}^3$).



But *all* the configurations with the density in the center greater than ρ_c , as well as the utmost inhomogeneous configuration, are to the left of the OV limit in M/R curve (point C in Fig.1). Hence, if GR remains true even in *such a strong* gravitational field then in nature there exist only strange stars as maximum compact *stationary* objects with *the surface* (i.e., with a finite z) but not with the event horizon. ("Charmed stars" do not exist in GR.) The utmost inhomogeneous configuration (point D in Fig.1) in which *all* quark (and lepton) generations would be "defrozen" is not realized either, according to GR - it is "eaten up" by black holes.

By reasoning of this section, we tried to make more concrete analogous speculations expressed by Alcock et al. (1986), in connection with their use of the same equation of state, independent of the number of particle flavors. Since if instead of the full expressions we use their limit (1), then, strictly speaking, everything that is said about quark masses, quark flavors, chemical potentials, number density of different flavors, density (ρ) at which "the appearance" of the next flavor occurs and even the use of some value α_c , all that is now only *interpretation* in the bounds

of perturbation QCD. The results of calculation of hydrostatically equilibrium configurations are determined in the end only by the value $4B/c^2$ or the energy density ϵ which the limit (1) can be used.

Certainly, one should try to understand why the results of such calculations differ in less than 4% from the results of calculations with the help of full expressions. That is why here we use the notion of "defreezing" of the next flavor, meaning first of all the fact that for this "new" flavor, at the given density, the condition $l > l_c$ begins to be fulfilled.

Besides, in so-called "full expressions" the used values of quark masses and the value of α_c are fixed. At the same time it is known that quark masses are measured at definite momenta. Numerical values which are usually used concern the distances between the quarks of the order of 10^{-14} cm, that in our case means a definite macroscopic density ρ . The less the distance between interacting quarks, i.e., the greater macroscopic density, the less quark masses can be. But this simply means that there must exist a dependence of α_c on ρ (more details will be in the next section). In other words

$$m_q(\alpha_c) = m_q[\alpha_c(\rho)] = m_q(\rho);$$

and then at calculations of hydrostatically equilibrium configurations only macroscopic density $\rho \geq 4B/c^2$ becomes really the basic parameter.

Then even in the case of the limit (1) it is not absolutely necessary to require that, for all ρ 's, the fulfillment of the equality $\alpha_c = 0$. For u - and d -quarks defreezed at $\rho > 4B/c^2$ at a given ρ , we can put $m_{u,d}(\rho) \rightarrow 0$, then $\alpha_c(\rho) \neq 0$, and to have possibility to use the perturbation theory the value $\alpha_c (2\alpha_c/\pi < 1)$ must be sufficiently small. At densities $\rho < \rho_s$ (8), the heavier quark s is present in plasma, but for it l is still rather small for its contribution into pressure could change essentially the equation of state. At densities $\rho > \rho_s$ in the full equations besides decrease of $\alpha_c(\rho)$, the value m_s can also tend to zero, that leads in turn to the disappearance of corresponding ("massive") terms in the full expressions. In the end we shall be even closer to the limit (1).

All this reasoning could be illustrated by corresponding calculations, but the matter is that nobody knows today the precise form of the dependence of $\alpha_c(\rho)$ and $m_q(\rho)$ and we can only guess (see the next section) how α_c will behave at high and superhigh ($\rho \gg \rho_{nucl}$) macroscopic densities. However, it may be the essence of differences between quark configuration calculation results in the bounds of asymptotic MIT bag model, i.e., with the help of Equation (1), and ones in the bounds of the Perturbative QCD model (Overgard and Ostgaard, 1991).

In conclusion of this section which has been dedicated to the pure quark configurations in GR, we emphasize once more that the compact object consisting of strange matter with the equation of state very close to (1) is the last opportunity of stable state after which only black holes with $z \rightarrow \infty$ follow.

3. QUARK-GLUON PLASMA IN GRAVIDYNAMICS

So, "quark stars" considered here as objects consisting of QGP including all possible flavors (u, d, s, c, b, t, \dots) and quark and lepton generations turn out to be unstable in GR. One can say that such objects must not exist in nature according to all versions of GR as well in which there are "frozen stars".

A "quark star" as a *limiting* (in several senses) stable object with the total mass M_Q with the QGP-bag surface ($z \neq \infty$) of the radius of $R_{QGP} \equiv GM_Q/c^2$ (inside the Shvarzshild sphere according to GR) can exist if we adhere to the dynamic, totally non-metric description of gravitational interaction.

In GD (unlike geometrodynamics - GR) the profile of the total energy density $\varepsilon(r)/c^2$ of analogous quark configuration does not terminate in a vacuum (see Fig.5). Around the macroscopic quark bag (QGP-bag) a fur-coat exists - the "gas" of virtual gravitons whose energy density has to be allowed for in the equation of state.

Here the *total* mass M_Q of the object entering the determination of $R_{QGP} \equiv GM_Q/c^2$ should be found in "long-wave limit" (like classic charge of electron), i.e., at $r \gg R_{QGP}$ or, in other words, the mass is determined by Newtonian rules. And if the object is described by 4-potential (4) from the paper by Sokolov (1992) with the radius of the bag $R=R_{QGP}$, then M_Q consists by *half* of the "coat" mass. Thus for the *total* energy Mc^2 of the quark star in GD (Q-star) one can write

$$\frac{1}{2} M_Q c^2 (\text{the bag with } R=R_{QGP}) + \frac{1}{2} M_Q c^2 (\text{the "coat" in vacuum}) = M_Q c^2.$$

Thus, the Q-star mass is determined by integration from its center ($r = 0$) and up to $r = \infty$. (This is an essential difference from the definition of mass in GR). Strictly speaking, this is the determination of mass of any objects in GD. By force of this fact in GD there is no event horizon that is ultimately the result of total refusal of geometric phenomenology constituting the base of GR. The bag surface can undergo some unselected sphere $r = 2GM_Q/c^2$ as a result of the relativistic collapse, but the *whole* gravitating object, including its gravitational "atmosphere" (or coat), can *never* be found under this "horizon".

For the object with limit achievable parameters, which a quark star if $R_{QGP} = 2GM_Q/c^2 \approx 10\text{km}$ seems to be in GD, the equation of state inside and outside the bag is the same:

$$P = \frac{1}{3} \varepsilon. \quad (10)$$

Inside the bag the total pressure is a sum of two ("partial") pressures. First of all it is the pressure

$$P_Q = \frac{1}{3} (\varepsilon - 4B).$$

Strictly speaking, this formula gives the pressure of degenerated Fermi-gas of free and massless quarks. Then the contribution of gluons (with some admixture of

gravitons?) in the limit under investigation is determined by the equation

$$P_G = \frac{1}{3} (4B). \quad (11)$$

Hereafter, we shall proceed from the assumption which is apparently true in the case of huge ($\rho \gg \rho_{\text{nuc1}}$) macroscopic densities in question. We consider that gluons which are free and non-interacting with each other (at the total density increasing to the center) can be distributed inside the bag with constant and positive density $4B$. The interaction of gluons becomes essential far from the center, maybe even near the very "wall" of the macroscopic bag, where (as a result of that) the energy-momentum tensor trace of massless gluon field becomes non-zero... We do not know so far at what distance from the bag walls it will occur, that is why we choose here "the simplest" limit case (11).

As a result of it, in the sphere $r = R_{\text{QGP}}$ the sum of pressures is equal to (10). The total energy density in the bag with $R_{\text{QGP}} \approx 10\text{km}$ decreases from the center according to the equation

$$\varepsilon(r) = \frac{4B}{c^2} R_{\text{QGP}}^2 r^{-2} \quad (12)$$

(Sokolov, 1991), if the equation of state inside the bag is taken in form (1). The QGP-bag in GD turns out to be connected only by colour forces ("wall" of bag), the gravitation inside the bag is "switched off" for such a *limiting* object which the quark star is. It can be assumed that in GD, in that case, QGP is in totally free (at $R = R_{\text{QGP}}$) unstressed self-bound state when there are no forces binding the bag besides colour ones. Then the distribution $\varepsilon(r)$ is here maximum inhomogeneous. It differs radically from an analogous case in GR at totally homogeneous distribution of density ($d\varepsilon/dr \rightarrow 0$) when $m \rightarrow 0$ also.

Outside the bag (in "vacuum") "a gas" still remains from virtual gravitons with the same equation of state (10). Positive energy density $\varepsilon = \theta^{00}(r)$ of the gravitational field falls here from the value $4B$ (at the boundary of the two "mediums": QGP - "vacuum") according to the law:

$$\theta^{00}(r) = \frac{4B}{c^2} R_{\text{QGP}}^4 r^{-4}, \quad (13)$$

which is connected with the fact that at distances $r \gg GM_Q/c^2$ the self-action of gravitons arises (scalar and tensor gravitons, tensor and tensor ones; Sokolov, 1990).

Of course, it is not excluded that in QCD-theory which would be more correct (than the bags model) the contribution of quarks and gluons in the total energy density (12) inside the QGP-bag could be distributed in an absolutely different manner than we assume here. But (most probable) these are such contributions of fermion and boson components at the huge ($\varepsilon/c^2 \gg \rho_{\text{nuc1}}$) density which increases towards the center ($\varepsilon \propto r^{-2}$), that as a result only such limit equation of state (10) conforming to the

rest of physics turns out to be true.

Strictly speaking, from the very beginning, the question in (1) was only on quarks since here there is no explicit contribution into pressure which corresponds to gluons. To make sure of that it is sufficient to look at the full equation of state before the pass to the limit (1). Thus, in the case of quark configurations in GR one cannot say about QGP: in such a plasma simply there are no gluons, if using the limit equation (1) as the equation of state. In GD we do deal with QGP since in GD we try to account *explicitly* for the contribution of bosons, at least in the case of asymptotically free gluons with the equation of state (11). The last can be justified apparently only in the case of macroscopic density of QGP (12) increasing to the center, when the constant of colour interaction decreases sufficiently quickly with the increase of ε from the walls of the bag towards its center. Below, in this section, it will be said that it is possible, in principle, at such a profile of $\varepsilon(r)$ as (12).

To obtain the *total* (observable) mass of such a quark configuration (unlike what was in GR) at the integration of ε/c^2 it is necessary to allow for the energy θ^{00} of the gravitational field itself. As a result, the mass of a quark star in GD can be expressed in terms of the energy density value at the boundary between QGP and "vacuum" in the following way:

$$M_Q = 6.64 M_\odot \left(\frac{2\rho_{\text{nucl}}}{4B/c^2} \right)^{1/2}; \quad (14)$$

and the same restrictions (5), which were mentioned above, fix the mass and the radius of the quark star in GD in the following limits:

$$6.21 M_\odot \leq M_Q \leq 7.25 M_\odot \quad (15)$$

$$9.16 \text{ km} \leq R_{\text{QGP}} \leq 10.69 \text{ km}.$$

The lowest value of the total mass of the configuration and, accordingly, the lowest value of the QGP bag radius (as was said above) follow from the condition that at the density (12) on the bag surface equals $\varepsilon/c^2 \approx 2.3\rho_{\text{nucl}}$ (3,5). This surface consists from strange self-connected (i.e., stable at $P_Q=0$) matter. Since ε increases towards the center of the bag and, consequently, all other flavors of quarks become defrozen, then in GD (unlike what was in GR) it is necessary to speak not about a strange star, but about a quark star with the strange surface, if the density on this surface does not exceed $\approx 2.3 \rho_{\text{nucl}}$.

If we assume that some other conditions of the type of (3) are possible, but at $P_Q \neq 0$, when at bigger densities on the QGP surface, already more massive quarks than s-quark become "defrozen", then according to (14) the masses of corresponding metastable quark configurations will be less than $6 M_\odot$ down to the values $M_Q \rightarrow 1.4 M_\odot$. But in any case, Witten's condition (3) at $P = 0$ fixes some maximum mass ($>6 M_\odot$)

of the most stable quark configuration in GR. Whether such a limit object exists allowing for physical conditions is another question. But such a limit can be a consequence in principle of GD and QCD, if GD gives more or less correct description of the strong gravitation.

Thus, as follows from a brief review of properties of compact objects, collapsars (Sokolov, 1991) - from masses and radii of such objects in binary systems such as Cyg X-1, A0620-00, LMC_X-1, LMC_X-3 - can be readily considered as "candidates" into quark stars of GD with the strange surface. Some properties of the quark star which could lead to corresponding observational manifestations are discussed in more detail in the quoted review. But apparently, the basic *observational consequence* confirming the version of GD \cup QCD suggested here could be indeed the existence of a selected mass value of collapsars (or "candidate into black holes") $\geq 6 M_{\odot}$. Since in GR there is no preferable mass values of black holes for all masses of "candidates" greater than OV-limit, then one will have to invent some astrophysical (e.g., evolutionary) arguments explicating the mass of a "typical" collapsar in such close binary systems.

Below we try to imagine how the "running" constant α_c of the strong interaction could depend on the parameters of the QGP bag under consideration, and, most importantly, what could be the dependence of this value on macroscopic density so that we could make at least the rough agreement between the asymptotic MIT bag model and the *interpretation* of results with the help of the perturbative QCD which we mention here rather often.

In Fig.5 we marked macroscopic densities above which the corresponding quarks are already in the state of deconfinement. The total density $\varepsilon(r)$ increases deep into the bag and, consequently at the approach to its center the quarks become compressed still tighter.

In other words, when the macroscopic density exceeds a certain level, the corresponding quarks are situated relative to each other and *interact with each other* at distances Δl less than $l \approx 10^{-13}$ cm. The same can be said about any particle of QGP. In particular, the known formula for the "running" constant α_c of colour interaction can be written in the form:

$$\alpha_c(\Delta l^2) = \frac{12 \pi}{(33-2n_f) \ln(l_c^2/\Delta l^2)}, \quad (16)$$

where n_f is the number of "defrozen" quark flavors, l_c , the radius of confinement, and Δl , the distance between two *neighbouring* strongly-interacting colour particles ($\Delta l \neq l$, l being the free-path length of quarks in QGP which can be much greater than l_c).

For example, if one demands that u -, d -quarks ($n_f=2$) could be considered almost free ($\alpha_c \approx 0.45$, see Haensel et al., 1986), it is necessary that the mean distances Δl , to which the quarks must be "compressed" in such QGP would be about

$0.25 \cdot l_c \approx 10^{-14}$ cm. One may think (see the previous section) that these are just the conditions ($\Delta l < l_c$, $\alpha_c < 1$) that are realized in plasma first for u - and d -quarks at the macroscopic density $\rho = \epsilon/c^2$, greater than the density at the boundary of the QGP-bag (i.e., greater than the density of phase transition in the QGP state).

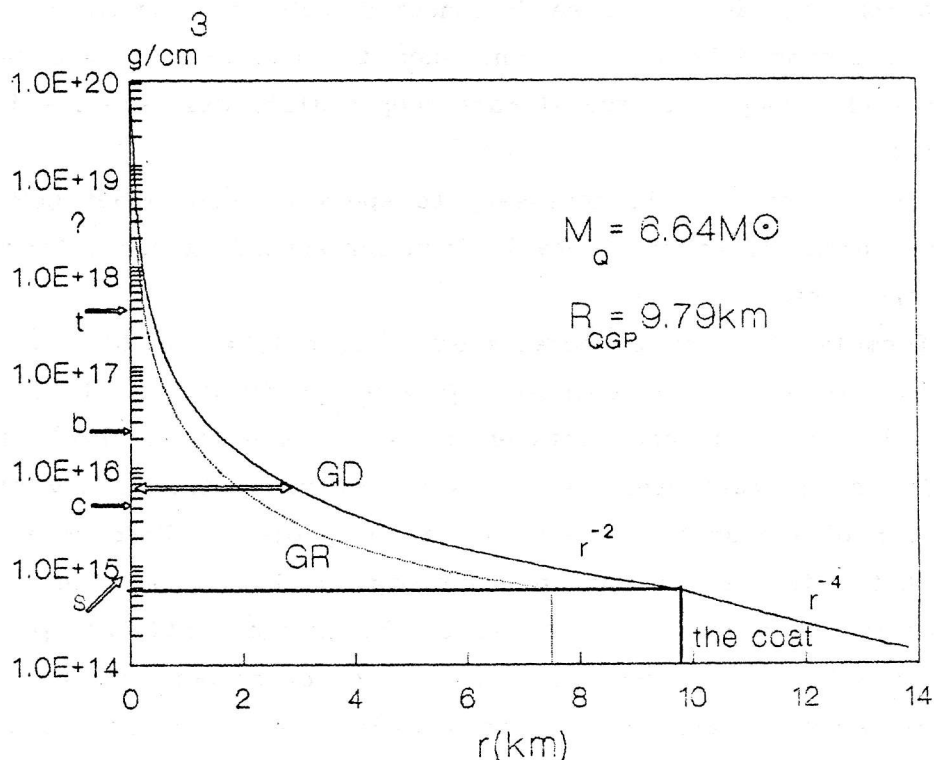


Fig.5. Density profile for a quark star in GD (solid line). Fat rectangle shows "the background" created by gluons (see the text) distributed homogeneously(?) in the bag with $R_{\text{QGP}} \approx 10$ km. "Vacuum" around the bag is filled by a "gas" of virtual gravitons (the furcoat) with energy density $\theta^0(r)$. The densities are indicated at which "the defreezing" of s, c, b, t, \dots quarks occurs. The arrow indicates the density at which the perturbative QCD vacuum must be totally restored ($\alpha_{\text{cm}} \approx 0.7$). The dotted line is the density profile of analogous quark configuration in GR for $4B/c^2 = 2\rho_{\text{nuc1}}$ ($B = 78.5$ MeV/fm³).

If we apply the idea of coupling constants depending on density (which is widely used in "standard" cosmology of Big Bang for our case of macroscopic bag) one can try to parameterize the macroscopic constant of strong interaction by the equation

$$\alpha_{\text{cm}}(\rho) = \frac{12 \pi}{(33 - 2n_f) \ln(\rho/4B/c^2)} \quad (17)$$

We emphasize here that ρ is macroscopic and, therefore, a certain mean density which does not exclude that microscopic fluctuations of density in volumes of $\sim (\Delta l)^3$ can, generally speaking, exceed ρ dozens of times. In particular, if we remember here the attempts of getting "hot" QGP on colliders in volumes of the order of several fm³, then the corresponding macroscopic density (here the macroscopic volumes of averaging

$\approx 1 \text{ cm}^3$ are also meant) will be simply zero. In that case there is no question about any gravitational effects which become essential only at big macroscopic masses. Here we mean cold catalyzed self-connected matter which must be the source of gravitational field.

On the other hand, the strong interaction which is realized here in big (macroscopic) volumes has also the character of macroscopic colour interaction, something like "colour gravitation" inside a huge self-connected QGP-bag. Formula (17) could describe just such a macroscopic interaction, when the exchange by gluons between the elements inside such a bag, situated at macroscopic distances relative to each other, becomes essential.

Thus here, in our case, it is necessary to speak already about QGP in astrophysical conditions which differ considerably from corresponding conditions available in experiments on the Earth.

Of course, formula (17) can be considered still only as an attempt of some rude extrapolation in the region of superhigh ($\rho \gg \rho_{\text{nucl}}$) densities. In particular, the calculation of Δl - mean distance between quarks at a given $\rho > 4B/c^2$, the mean free path l and also other parameters of QGP will demand further study of microscopic properties of such plasma as it is made for ordinary plasma. Here we are to meet the same problems that exist in QGP and in quark bag models (in particular). Especially since the notion of the macroscopic QGP-bag can be used *directly* at $\rho \propto r^{-2}$. Really, by use of formula (12), the macroscopic constant α_{cm} of colour interaction (of macroscopic volume elements situated at macroscopic distances from each other) inside the QGP-bag can be expressed in terms of r - the distance from the bag center - in the following way:

$$\alpha_{\text{cm}}(r^2) = \frac{12 \pi}{(33-2n_f) \ln(R_{\text{QGP}}^2/r^2)} \quad (18)$$

Then, the colour confinement in the gigantic bag of R_{QGP} radius is provided here in the same way as it was in the model of *homogeneous* (in density) MIT-bag with the "radius" l_c in formula (16). In particular, the values α_c and α_{cm} must be in approximately the same relation as the value $|\varepsilon_v|$ - the energy density of QCD-vacuum (obtained from the analysis of sum laws) and the value of the constant B (which is connected with hadron mass spectrum). It naturally follows from the fact that in (17) we actually use directly the model of quark bags.

If, finally, some *definite* value of density at the boundary of the QGP-bag in accordance with the restrictions (5) is chosen, then in evaluation of value α_{cm} one can use the equation

$$\alpha_{\text{cm}}(\rho) = \frac{12 \pi}{(33-2n_f) \ln(\rho/2\rho_{\text{nucl}})} \quad (19)$$

Equations (18) and (19) should be considered here only as an attempt to make the

interpretation by means of perturbative QCD, to which we resorted in this and previous sections, agree with the asymptotic MIT bag model. Of course, such equations can be an approximation as the "initial" approximated equation (16) itself. The most probable, the more precise dependence $\alpha_{cm}(\rho)$ will lead to even more quick decrease of colour forces in the direction from the bag wall towards its center. In the end we shall have to use only limit (1) for the cold, catalyzed QGP. It may be, at this limit, the difference between MIT-bag and QCD approaches will decrease or disappear at all. One can also assume that in that case there can realize such physical conditions when at the disappearance of the energy-momentum tensor trace of massless gluon field inside such a QGP bag we can speak already about a *classical limit* of QCD.

But one way or another, at consideration of macroscopic QGP (i.e., QGP in astrophysical conditions) and the use of QCD, an absolutely definite dependence for $\alpha_c(\rho)$ or $\alpha_{cm}(\rho)$ will be needed for sure. Below we show by corresponding estimations the fact that an approximate equations (19) suggested above does not contradict considerably to "the standards" of QCD.

If we consider that the perturbative vacuum is restored at such macroscopic densities ρ_{QCD} that the relation of ρ_{QCD} to the density on the bag border is the same as the relation (4) between $|\epsilon_v|$ and B_{MIT} ,

$$\rho = \rho_{QCD} \approx 2\rho_{nucl} \cdot \frac{|\epsilon_v|}{B_{MIT}} \approx \frac{2\rho_{nucl}}{0.13} = 43.07 \cdot 10^{14} \text{ g/cm}^3 \quad (20)$$

(when four quark flavors are defrozen, see Fig.5), then (19) yields $\alpha_{cm} \approx 0.74$ at $\rho = \rho_{QCD}$. It is close indeed to the QCD-value $\alpha_c^{QCD}(1 \text{ GeV}) = 0.7$. At the same time, for strange matter ($n_f = 3$) at macroscopic densities, less than ρ_{QCD} ($\rho \approx 10^{15} \text{ g/cm}^3$), the macroscopic constant of colour interaction turns out to be equal to $\alpha_{cm} \approx 2.4$. It is close to the value of $\alpha_c^{MIT} = 2.2$ determining the mass splitting of hadron multiplets (Bogolyubov, 1968; De Grand et al., 1975).

Thus both from equation (19) and from estimation (20), it follows that QCD-vacuum (perturbative vacuum) in strange matter is not restored yet ($\alpha_{cm} \approx 0.87$), which agrees with calculations in the bounds of GR carried out by Kondratyuk et al. (1990). As follows from Fig.5 and equation (19), (20) the perturbative vacuum is totally restored in the interior of a QGP-bag of a quark star in GD at the depth $R_{QGP} - r \approx 7 \text{ km}$, i.e., in the case of the most stable (limit) quark configuration with the bag whose surface consists of strange matter.

4. CONCLUSION

From equation (19) it follows, in particular, that in the very center of the QGP-bag with $R_{QGP} \approx 10 \text{ km}$ (i.e., at $r = 10^{-13} \text{ cm}$) the macroscopic constant of colour forces is

only about 3 constants of electromagnetic interaction. The density ρ here must be about $5.4 \cdot 10^{52} \text{ g/cm}^3$, and the mass (in $r=1 \text{ fm}$ sphere) must equal $7 \cdot 10^{14} \text{ g}$. Of course at mutual distances between QGP particles much less than l_c (10^{-17} cm) and, correspondingly, at densities $\gg 10^{16} \text{ g/cm}^3$ the constants of all the three fundamental interactions (strong, weak, and electromagnetic ones) must in the end become indistinguishable from each other.

Thus, in the interiors of a GD quark star - a stationary stable object with density increasing to the center according to the law $\varepsilon(r) \propto r^{-2}$ - just the physical conditions can be realized under which all the interactions unite in one fundamental interaction. For sure, the constants of weak and electromagnetic interactions inside the QGP-bag of the quark star can be also expressed in terms of *macroscopic* densities. And it means that the ideas of *Grand Unification* of all interactions could be tested without resorting to cosmology of Big Bang, but studying *the same* physics of *superhigh densities* (and "cosmomicrophysics"), observing compact objects (bright X-sources, γ -ray bursts, remnants of supernova explosions, etc.) of stellar masses.

Of course, it should be admitted here that GR describes erroneously the strong gravitational field of such objects. But ultimately, if we abandon the conviction (a priori) of *absolute* correctness of GR and do not forget that only a *sufficiently* complete experimental (observational) study of strong gravitational fields can affirm or refute this conviction, then all the preceding discussions can be considered as a possible alternative to black holes of GR.

We recognize that sometimes statements of certain things looks schematic here. The matter is that many QGP properties and QGP itself is only a hypothesis, although a hypothesis which follows naturally from the experiments on colliders and from the theory of quarks and leptons. But even now from all what has been said, it is clear that a consistent *direct* allowance for localizable positive (like in the case of all other gauge fields) energy of gravitational field changes completely the collapsar physics. In particular, one of the observational (experimental) arguments in favour of such or similar physics would be the existence of some selected value for the collapsar mass (or for "the candidates in black holes" of GR). Proceeding from the theoretical scheme developed here, we consider that the collapsar - a compact object with the mass exceeding, certainly, the pulsar mass (or OV-limit in GR) - can be identified with the limit quark configuration in GD whose mass is $\geq 6 M_{\odot}$.

REFERENCES

- Alcock C., Farhi E., Olinto A.: 1986, *Ap.J.*, **310**, 261.
Benvenuto O.G., Horvath J.E.: 1989, *Mon.Not.R.Astron.Soc.*, **241**, 43-50.
Bogolyubov P.N., 1968, *Ann.Inst.Henri Poincare*, **8**, 163.
Chodos A., Jaffe R.L., Johnson K., Thorn C.B., Weisskopf V.: 1974, *Phys.Rev.*, **D9**,

- 3471.
- Collins J.C., Perry M.: 1975, *Phys., Rev. Lett.*, **34**, 1353.
- De Grand E., Jaffe R.L., Johnson K., Kiskis J.J.: 1975, *Phys. Rev.*, **D12**, 2060.
- Emel'yanov V.M., Nikitin Yu.P., Vanyashin A.V.: 1990, *Fortschritte der Physik*, **38**, 1.
- Fock I.A.: 1961, *Theory of Space, Time and Gravity*. Nauka, Moscow.
- Haensel P., Zdunik J.L., Schaeffer R.: 1986, *Astron. and Astrophys.*, **160**, 121.
- Haensel P.: 1987, *Prog. Theor. Phys. Suppl.*, No. 91, 268.
- Kondratyuk L.A., Krivoruchenko M.I., Martemyanov B.V.: 1990, *Pis'ma v A. Zh.*, **16**, No. 10, 954.
- Krivoruchenko M.I.: 1987, *Pis'ma v ZhETF*, **46**, 5.
- Novikov V.A., Shifman M.F., Weinstein A.I., Zakharov V.N.: 1981, *Nucl. Phys.*, **B191**, 301.
- Overgard T., Ostgaard E.: 1991, *Astron. and Astrophys.*, **243**, 412.
- Ryskin M.G.: 1990, *Journ. Nucl. Phys.*, **52**, 219.
- Shapiro S.L., Teukolsky S.A.: 1983, *Black Holes, White Dwarfs and Neutron Stars*. N.Y.: by John Wiley & Sons, Inc.
- Sokolov V.V.: 1990, *Problems on High Energy Physics and Field Theory (Proc. of the XII workshop, Protvino, July 3-7, 1989)* Ed.: V.A. Petrov, Moscow.: Nauka, 45.
- Sokolov V.V.: 1991, *Problems on High Energy Physics and Field Theory (Proc. of XIII workshop, Protvino, July 9-13, 1990)*. Ed.: S.M. Troshin, Moscow.: Nauka, 1991, 39.
- Sokolov V.V.: 1992, *Astron. & Space Sci.* **197**, 179.
- Witten E.: 1984, *Phys. Rev.* **D30**, 272.
- Zel'dovich Ya.B., Novikov I.D.: 1971, *Theory of gravity and star evolution*, Moscow.: Nauka.