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SCIENTIFIC RESEARCHES

Evolution of magnetic fields of white dwarfs

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Abstract. Based on photometry and parallax data temperature estimates of 1225 nonmagnetic and 49 magnetic white dwarfs as well as mass estimates of 1019 nonmagnetic and 35 magnetic white dwarfs have been obtained. The frequency of magnetic degenerates has been found to be strongly dependent on the temperature of these stars. We have confirmed the first conclusion of Liebert and Sion (1979) that the frequency of magnetic white dwarfs is higher among cool stars than among hot stars. Possible effects of observational selection are discussed and taken into account. As the degenerates cool down, both the number of magnetics among them and their average magnetic field strength increase with a time scale $t \approx 2-3$ Gy and reach maximum at a photosphere temperature $T \approx 6000-8000$ K. With further cooling down the average magnetic field strength is observed to decrease which is possibly caused by the field decay. This effect has been found, however, marginally insignificant. An assumption has been made that the function of evolution of magnetic fields in white dwarfs $B(B_0, t)$ does not depend (or only slightly depends) on the initial magnetic field strength B_0. The observed magnetic field evolution is discussed within the frames of two scenarios: 1) the variation of degenerate magnetism follows the evolution of electric conductivity inside a star under the condition that electromotive force of nonelectromagnetic origin exists; 2) the magnetic fields, "buried" at degenerate formation diffuse outward. Magnetic white dwarfs have been found to be more massive than the average white dwarf. This confirms the result of Liebert (1988). The mean mass of a magnetic white dwarf is $M \approx 0.9M_\odot$, which is about $\approx 0.25M_\odot$ larger than that of a nonmagnetic. It has been revealed that the frequency of magnetic stars among the most massive degenerates is nearly an order of magnitude higher than the average. Magnetic white dwarfs in themselves are not a homogeneous class of stars. They represent two types of objects: a population of white dwarfs of "ordinary" masses whose magnetic fields $B_s < 100$ MG; and a population of "ultramassive – ultramagnetic" white dwarfs with masses $\gtrsim 1.1M_\odot$ and magnetic fields $B_s > 100$ MG. Among the stars of the latter population the magnetic field decay observed with a time scale of ≈ 3 Gy. Such stars amount to about 20 % among magnetic white dwarfs.

Key words: stars: white dwarfs: magnetic fields – evolution

1. Introduction

A study of magnetism of white dwarfs is extremely important for it throws light on the problems of origin of magnetic fields in stars, magnetism and internal structure of the main sequence stars, formation and structure of white dwarfs, evolution of binary stars, etc. Magnetic fields in white dwarfs were first recorded by Kemp (1970) more than 20 years after the discovery of magnetic fields in early–type stars (Babcock, 1947) and the first investigations of magnetism of white dwarfs (Blackett, 1947). The first detected magnetic white dwarfs turned out to be cool stars (Greenstein et al., 1971; Greenstein, 1974). That is why it was concluded that magnetic stars are widespread among cool white dwarfs as compared to hot white dwarfs. It was suggested that as a degenerate cools down, a deep convective zone arises in the star, which carries outward internal magnetic fields of high intensity. However, subsequent discoveries of hot magnetic degenerates were at variance with these conclusions. The next step was made by Liebert and Sion (1979) who found magnetic white dwarfs to occur more frequently among cool stars than among hot stars. They could study only 13 magnetic white dwarfs from a sample of 450 white dwarfs known at that time. Probably for this reason the significance of their result was not high.

It is important that Liebert and Sion (1979) found the number of magnetic stars among white dwarfs to decrease with increasing distance to them. This could be a consequence of the reduced frequency of magnetic white dwarfs with increasing temperature of stars, and alternatively, this may result from apparent observational selection effects. Indeed, it is much more difficult to detect the magnetic field in faint stars than in bright stars. Somewhat later the matter of relationship between the magnetism of white dwarfs and their temperature received a detailed discussion in the paper by Angel et al. (1981). The authors came to the conclusion that the enhanced apparent frequency of magnetic degenerates among cool stars is not significant.

Subsequent discoveries of hot magnetic white dwarfs with X–ray telescopes and also in the surveys of blue–excess objects (e.g. Reimers et al., 1996) increased sharply the number of the known hot magnetic white dwarfs, and that is why the matter of enhanced frequency of magnetic white dwarfs as they cool down was no longer dealt with. Probably for all these reasons almost no investigations of evolution of magnetic fields in white dwarfs from observational data were carried out. Nevertheless, the theory of this matter is being developed quite successfully (Wendell et al., 1987; Muslimov et al., 1995). Recently Putney (1997) has published results of spectropolarimetric observations of DC white dwarfs. It follows from the observations that the share of magnetic white dwarfs among DC stars is about 15 %, while the frequency of magnetic stars among DA is about 2 %. Since the temperature of DC degenerates is on average lower than that of DA stars, these results are not contrary to the suggestion that magnetic white dwarfs may be more frequent among cool stars than among hot stars.

At present data are available on the magnetic fields of 53 white dwarfs (Schmidt and Smith, 1995; Jordan, 1997; Fabrika and Valyavin, 1998). In the previous papers (Valyavin and Fabrika, 1997; Fabrika et al., 1998) on the basis of the analysis of the present–day data we have found that the frequency of magnetic white dwarfs and the mean magnetic field strength do increase as they cool down, reach a maximum at a temperature of about 10000 K. In the case of cooler stars we are likely to observe even the process of magnetic field decay. In the present paper we discuss in more detail the observed evolution of surface magnetic fields of white dwarfs, i.e. the behaviour of their magnetic fields versus temperature and age. On the basis of atmosphere models of white dwarfs, their evolutionary sequences, photometry and parallax data we find temperatures, masses and ages of all spectroscopically classified white dwarfs, for which photometry data are available. We discuss and take into acount the main observational selection effects. The behaviour of magnetic white dwarf frequency depending on their temperature, mass and age is analyzed.

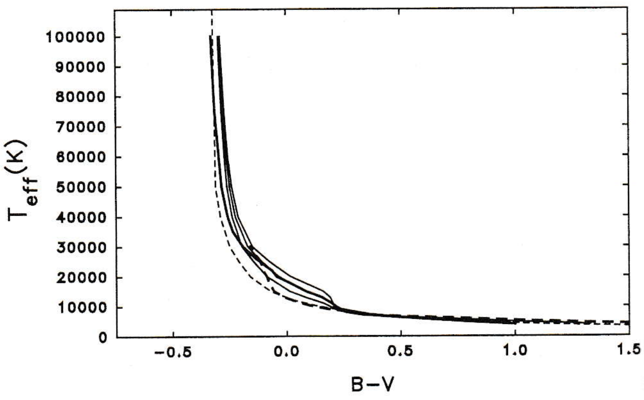

Figure 1: *The calibrations (B–V) — T_{eff} for published models with different lg g, hydrogen and helium abundances and for black–body spectrum.*

2. Effective temperatures of magnetic and nonmagnetic white dwarfs

In order to find the frequency distribution of magnetic white dwarfs with temperatures, one needs first of all a sample of white dwarfs with estimated effective temperatures, which contains a complete subsample of magnetic white dwarfs. To reliably determine the frequency of magnetic stars in different temperature intervals, the sample must be large enough. There are a number of papers with estimates of effective temperatures of nonmagnetic white dwarfs (for instance, Shipman, 1979; Koester et al., 1979; Bergeron et al., 1992). The most accurate estimates of the temperature can be made using the atmosphere model methods. These estimates are made for the brightest and well studied stars. By the present time data on the temperatures of about 200 such stars have been published (e.g. Koester et al., 1979; Bergeron et al., 1992). Unfortunately, this sample of white dwarfs is not large enough for our objectives. In this connection, based on the photometry data of the white dwarf catalogue by McCook and Sion (1987; 1999) we estimate the effective temperatures for as many of nonmagnetic white dwarfs as possible. These estimates are less accurate, but nevertheless, they allow an essentially large sample to be made up. In the catalogue broad band UBV photometry data are best represented. Koester et al. (1979) reported results of analysis of model white dwarf spectra with hydrogen atmospheres for different photometric systems. The model they use allows for the blanketing effect and convection. Similar calculations are given by Bergeron et al. (1995a).

Examples of the calculations (B–V) — T_{eff} are shown in Fig. 1. Here the thin solid lines are the model relationships given by Koester et al. (1979) for lg g = 7, 8, 9. The bold dashed and solid lines are the relationships obtained by Bergeron et al., (1995a) for lg g = 8 and for different hydrogen and helium abundances. The thin dashed line represents the black–body colour indices. At low effective temperatures the deviation of the model relationship from the black–body decreases since hydrogen lines disappear and the blanketing becomes less effective. It is seen from the figure that the error of temperature determination connected with the uncertainty in lg g and chemical composition may reach 30 % at B–V$\approx$ 0. The uncertainty in chemical composition and lg g around $B - V < -0.1$ makes it possible to estimate the temperature with about the same accuracy. In this region of temperatures accurate B–V data are very important. At $B - V > 0$ the temperature determination accuracy is much better.

The accuracy of temperature evaluation could be improved by using different photometric systems and obtaining the mean estimate. Besides the colour indices B–V and U–B the catalogue (McCook and Sion, 1999) provides also the Stromgren colour indices u–b and b–y and that of multichannel spectrophotometry g–r. For these data Bergeron et al. (1995a) give the model calibrations the colour index — T_{eff} at lg g = 8. Our analysis has shown that the use of the entire photometry available in the catalogue leads to improvement of the accuracy in T_{eff}, for hot white dwarfs in particular. The errors of temperature estimates in this case do not exceed 30 % and for effective temperatures less than 10000 K the error is not larger than 10–15 %. Based on these relationships we have obtained a sample of 1274 white dwarfs with temperatures estimated to the accuracy indicated above.

Schmidt and Smith (1995), Jordan (1997), Fabrika and Valyavin (1998) and a number of other authors give a summary of the effective temperature estimates for magnetic white dwarfs. The estimates may be classified in two groups. The first includes stars that have been studied by the atmosphere model methods with allowance made for the magnetic field (for instance, Achilleos and Wickramasinghe, 1989; Jordan, 1992). The second comprises stars whose effective temperatures have been estimated from fitting of their spectra with the black–body one or from their colour indices. Because of the considerable distortion of energy distribution in the spectra of magnetic white dwarfs the temperature estimates in the latter case can be less accurate. It is essential to analyze the error in the effective temperatures of magnetic white dwarfs determined from photometry data.

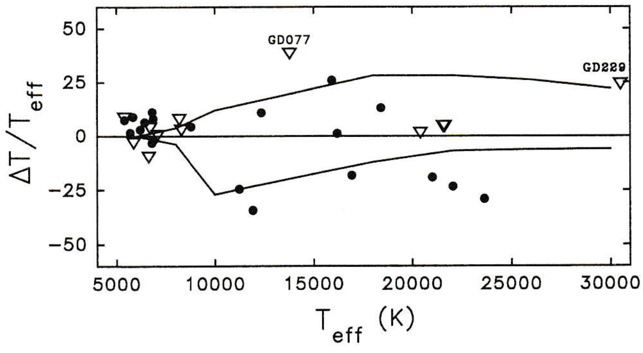

Figure 2: *The temperature estimate errors* $(T_{eff} - T^*_{eff})/T_{eff}$ *of magnetic white dwarfs. Individual values of* T^*_{eff} *are from detailed spectra modelling with taking into account magnetic field (circles) and from black–body fitting or these are the rough estimates from spectrum type (triangles).*

The results of such an analysis are presented in Fig. 2. The effective temperatures of magnetic degenerates estimated with the photometric calibrations obtained for the nonmagnetic case are plotted on the horizontal axis. The figure presents $\Delta T/T_{eff} = (T_{eff} - T^*_{eff})/T_{eff}$, where T^*_{eff} is the effective temperature of magnetic white dwarfs given by other authors. In particular, circles mark the stars whose T^*_{eff} has been estimated from the detailed spectra modelling of magnetic white dwarfs taking into account the known properties of polarized radiation (Jordan, 1992; Achilleos and Wickramasinghe, 1989; Schmidt et al., 1992). Triangles indicate the stars whose T^*_{eff} has been found by different authors either from black–body fitting, or these are the broad-band photometric "nonmagnetic" estimates or the rough estimates from spectrum type (Fabrika and Valyavin, 1998 and references therein). It can be seen from the figure that within an accuracy of 30 % the detailed model estimates of magnetic white dwarf temperatures agree with the estimates obtained on the basis of colours and atmosphere models with no regard for magnetic field.

To estimate the error in temperature caused by the disregard of magnetic field, we have computed a model grid of simple synthetic spectra for different temperatures and lg g = 8 taking account of the magnetic field. The modelling was performed on the basis of the programme ATLAS (Kurucz, 1970). The wavelengths and the oscillator strengths of the Zeeman hydrogen components of Hα, Hβ and Hγ were taken based on the calculations of Kemic (1974), Wunner et al. (1985), Fassbinder and Schweizer (1996), Ruder (1977), Fassbinder (1997). The unknown Stark broadening constants of the hydrogen components were chosen so that the full line at a sufficiently weak magnetic fields has the same profile and intensity as the line at zero magnetic field. The hydrostatic equilibrium in the atmosphere was calculated with no account of magnetic field. Thus we have calculated the spectra of magnetic white dwarfs with purely hydrogen atmospheres of different effective temperatures, $\lg g = 8$, in the magnetic field range from 1 MG to ≈ 50 MG. The synthetic spectra obtained were convolved with the different photometric systems presented in the catalogue by McCook and Sion (1999).

As a result of the Zeeman component shifts and appearance of new components as the magnetic field increases, the spectrum of a star and its colours change. So a certain effective temperature corresponds to each value of the colour index, and the temperature is different for different magnetic field strengths. Let T^{**}_{eff} be the temperature of the atmosphere with $\lg g = 8$ and the magnetic field strength B_s. From the colour indices corresponding to the model spectrum, T_{eff} value can be found by applying our calibrations to the nonmagnetic case and $\lg g = 8$. As a result one can determine the region of the $\Delta T/T_{eff} = (T_{eff} - T^{**}_{eff})/T_{eff}$ corresponding to different magnetic field values. The upper and lower lines in Fig. 2 correspond approximately to the limiting $\Delta T/T_{eff}$ values obtained by the modelling described above. They roughly confine the region of this value variations. The horizontal line represents $\Delta T/T_{eff}$ for $B_s = 0$.

With increasing magnetic field the value of $\Delta T/T_{eff}$ begins to decrease up to $B_s \approx 10$ MG (the lower curve in the figure). For larger magnetic field values $\Delta T/T_{eff}$ already increases, and from the field strength ≈ 20 MG it becomes positive. This is connected with the leaving the B band of a considerable number of strong components of $H\gamma$ and $H\beta$ lines and with the appearance of the $H\alpha$ and $H\beta$ components in V band. The value of $\Delta T/T_{eff}$ in our models nearly ceases to change with increasing the surface magnetic field at $B_s \geq 50$ MG. Since in the model spectra we take into account, in fact, only the contribution of the split line components, and the Zeeman components at $B_s > 50$ MG are separated by quite large distances ("mix"), even at very high magnetic field strengths the value of $\Delta T/T_{eff}$ might be close to the upper curve in Fig. 2. Thus the upper curve, which corresponds to 50 MG, may be treated in our model spectra as the upper limit of the $\Delta T/T_{eff}$ value.

The model curves can correctly be compared only with the data designated by the circles in Fig. 2, as the temperature T_{eff} of these stars was estimated on the basis of detailed atmosphere models. The two model curves outline quite well the region occupied by the circles up to effective temperature $T_{eff} \approx 20000$ K. This indicates that in this range of temperatures the model described can be used as a test for temperature estimates obtained without detailed modelling of atmospheres with a magnetic field.

The blanketing which is caused by Balmer line components shifted to UV region is not at all taken into account in our model. The effect is important at high temperatures and magnetic field strengths. The blanketing results in a depression in the UV region (the "ultraviolet catastrophe") and an increase in intensity of radiation in the optical range. This effect has an influence in the brightness in visible, making a star brighter, i.e. it must be taken into account when calculating bolometric corrections. This effect may influence the colour indices. Most authors attempt to allow for the blanketing in their models (for instance Jordan, 1992), that is why the disagreement between our simple model and individual "refined" estimates in the region $T_{eff} > 20000$ K may be associated with the blanketing. It can, however, be seen from Fig. 2 that the disregard of the blanketing is not important up to temperatures of 20000 K. The B–V index, however, is likely to be the least affected, since a temperature of radiation we observe in visible region must represent the true temperature of the photosphere because of thermalization of radiation in the photosphere.

Thus the analysis of the data presented in Fig. 2 makes it possible to illustrate the error caused by the disregard of Zeeman line components or the blanketing, on the one hand, and on the other hand, to isolate stars whose temperature estimates are questionable. GD 077 ($B_s \approx 0.9$ MG) is among such stars. Its temperature T^{*}_{eff} has been estimated from spectrum (DA 5, McCook and Sion, 1987) and is likely to be underestimated. The star GD 229 has the strongest magnetic field of all known magnetic degenerates, so it should be discussed in more detail. Its effective temperature estimates range from 16000 K (Green and Liebert, 1981) to 23000 K (Schmidt et al., 1996). The surface magnetic field of GD 229 is an order of magnitude higher than the upper limit of the known model grid computed, that is why its temperature may actually be somewhat different. Since the Zeeman calculations presently available permit UV opacity to be more or less correctly taken into account only to magnetic field values of about a hundred of MG, the last note can be referred to all white dwarfs with $B_s > 100$ MG.

The analysis presented, despite being rather of qualitative than quantitative character, is quite illustrative of the problem of effective temperatures in magnetic white dwarfs. It follows from Fig. 2 that even when the temperature is obtained with nonmagnetic calibrations, the error will be not larger than 30 %. Such an accuracy is quite acceptible to study magnetic white dwarf frequencies. Temperature intervals in which the data may be averaged should be chosen on the basis of the temperature determination accuracy. This allows all estimates of effective temperatures of white dwarfs available presently and also our estimates in the cases where the published temperatures are rather crude to be used.

3. Relationship between magnetism of white dwarfs and temperature

The observed frequency of magnetic white dwarfs P_m is determined by the ratio of the number N_m of known magnetic white dwarfs to the total number N of known magnetic and nonmagnetic stars of the same temperature interval. Let us define the observational selection coefficients $K(T_{eff})$ for nonmagnetic and $K_m(T_{eff})$ for magnetic degenerates. Then the frequency of magnetic white dwarfs is $P_m(T_{eff}) = K_m(T_{eff})N_m(T_{eff})/K(T_{eff})N(T_{eff})$.

Magnetic white dwarfs are generally identified merely from spectrum. Spectral line intensities depend on temperature and magnetic field strength. With magnetic splitting of a spectral line, its total equivalent width slightly grows, while the intensity of individual components drops. When the magnetic field is 5 MG $< B_s <$ 50 MG the spectral line components are deep enough, and the spectrum can reliably be identified. In some relatively bright objects the lines are well visible with larger magnetic fields too. The magnetic splitting becomes conspicuous in moderate dispersion spectra beginning with a surface magnetic field of 4–6 MG. Since magnetic objects can be reliably recognized from spectra with a moderate spectral resolution, we believe that the selection of identification of magnetic stars in the interval $\approx$5–100 MG from spectra is minor. Uncertainties here are connected with continuum–like spectra of DC stars, whose magnetic nature can be proved by polarimetric observations, and with white dwarfs having peculiar spectra. The fraction of such objects does not exceed 10–15 %. As a rule, these include stars with $T_{eff} < 6000 - 8000$ K, or very hot stars, when the hydrogen lines are considerably weakened. In this case the discovery of magnetic white dwarfs is impeded. That is why with very cold and very hot stars we may underestimate the frequency of magnetic white dwarfs.

In the region of temperatures $T_{eff} \approx$ 8000–20000 K there are no apparent reasons to think that we underestimate the number of stars with magnetic fields larger than a few MG. Therefore, for the analysis the data selection coefficients $K(T_{eff})$ and $K_m(T_{eff})$ will be considered equal, and the frequency is $P_m(T_{eff}) = N_m(T_{eff}/N(T_{eff})$ in this case.

It is important to consider the observational selections as dependent on distance. In the paper (Fabrika and Valyavin, 1998) we treat in detail the matter of equality of the "magnetic" and "nonmagnetic" selection coefficients, it is shown therein that the data could be considered as selection–independent for nearby and, therefore, the brightest and best studied white dwarfs located no further than 25 pc. For this reason now we will analyse the frequencies both among all magnetic stars and among nearest stars separately.

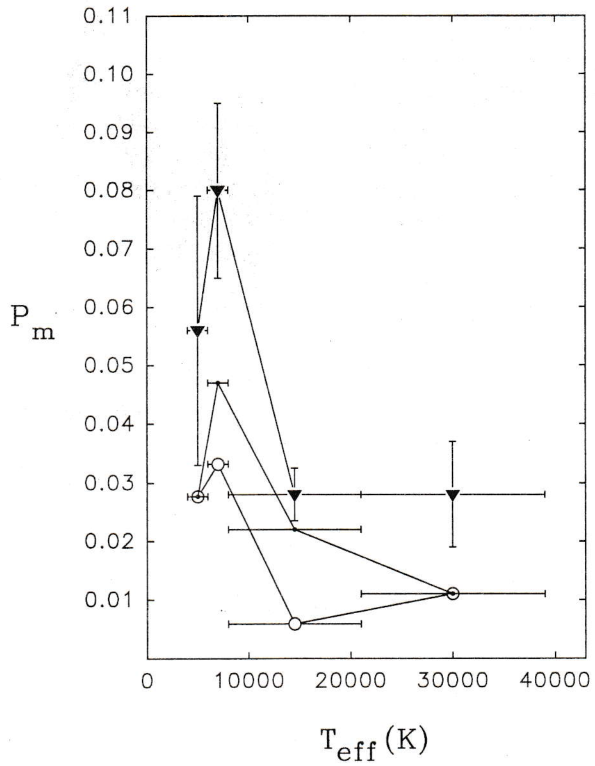

Figure 3: *The frequency of magnetic white dwarfs versus effective temperature. Circles denote the frequency of stars with magnetic fields 1–10 MG (16 stars), dots — from 10 MG and above (27 stars). Triangles are the estimates of $P_m(T_{eff})$ for all stars with fields over 1 MG.*

The foundation of our data is the catalogue by McCook, Sion (1987) as well as its new electronic version (McCook, Sion, 1999). Spectrophotometric and astrometric data for more than 2000 spectrally classified white dwarfs are presented there. The catalogue contains 1274 white dwarfs (38 of them are magnetic) for which one can estimate the effective temperature in a common way. This sample is added by 10 magnetic white dwarfs from the same catalogue, for which photometry data are lacking, but the temperature was estimated from spectra.

In Fig. 3 the frequency of magnetic stars versus effective temperature is shown. The temperature intervals are selected so that the mean temperature error would be markedly smaller than an interval and possible systematic errors would be smoothed to the utmost. To illustrate observational selections as dependent on magnetic field strength, we present the frequency $P_m(T_{eff})$ for magnetic white dwarfs with different B_s values. The circles in the figure denote the frequency of stars with magnetic fields 1–10 MG (16 stars), the dots — from 10 MG and above (27 stars). The triangles are the estimates of $P_m(T_{eff})$ for all stars with fields over 1 MG. The number of such stars is 43 since 3 white dwarfs have magnetic fields below 1 MG (Schmidt and Smith, 1995), two stars have a temperature $T_{eff} \geq 40000$ K and are not considered. The horizontal bars correspond to the temperature intervals. The vertical ones show the errors in $P_m(T_{eff})$. The errors are defined by the "magnetic" and "nonmagnetic" samples in given temperature intervals. They were derived (r.m.s.) by Monte Carlo simulations. In order not to crowd up the figure, the error bars are given only for the frequencies of stars with magnetic fields greater than 1 MG.

In the three versions of $P_m(T_{eff})$, it is seen to increase with decreasing effective temperature. Comparison of the frequencies of white dwarfs with magnetic fields over 1 MG in two intervals 6000–8000 K and 8000–21000 K by the Student's criterion shows unequality of these quantities with a confidence level no less than 0.995. Before drawing the conclusion on the significance of rising frequencies with decreasing temperatures, the observational selections should be discussed once again.

It is seen that the three plots in Fig. 3 are similar, however the white dwarfs having fields of 1–10 MG are observed to be markedly deficient in the temperature interval 8000–21000 K. At these temperatures the hydrogen lines are the most intensive and broad. The Zeeman splitting of such lines in magnetic fields less than 4–6 MG may well prove to be unnoticed. Stars with surface fields ≥ 10 MG are free from this selection.

It is apparent that among cool stars an appreciable part of magnetic white dwarfs will be missing in the sample since the intensity of their spectral lines is very low. Besides these stars are essentially fainter than hot stars and their spectral analysis presents different problems. The magnetic nature of these stars often needs to be confirmed by polarimetric measurements. This selection when taken into account will merely cause the frequency of cool magnetic stars to rise, i.e. increase the amplitude of the effect under discussion. Hot white dwarfs, both magnetic and nonmagnetic, are basically studied in the literature. It is obvious that this is due to the observational selection since hot stars are brighter. It is for this reason that hot stars are more numerous among the known magnetic white dwarfs (Fabrika and Valyavin, 1998). In contrast, we have discovered that the real frequency of cool magnetic stars is greater than that of hot magnetic stars. It is vital to assess how many cool degenerate stars one can miss in the present-day surveys and how this may influence the shape of the relation displayed in Fig.3. This can be judged from a sample of bright and well studied stars in the closest vicinity of the Sun. Let us isolate the white dwarfs separated from the Sun by no more than 25 pc (Fabrika and Valyavin, 1998). The open circles in Fig.4 indicate the frequencies of magnetic white dwarfs (a total of 14 stars) among these nearby stars. To get the most representative sample, only one interval for hot stars, $T_{eff} = 8000 - 40000$ K, has been chosen. By the Student's criterion the maximum and minimum frequencies are different at a confidence level greater than 0.9995 level. The sample of nearest stars is most free from selections, that is why we can consider proved the effect of increasing the frequency of magnetic white dwarfs with decreasing temperature.

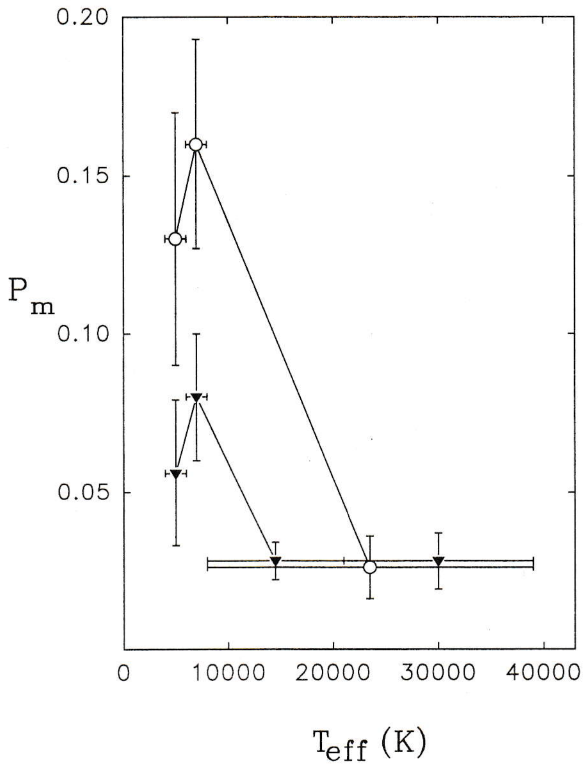

Figure 4: *The frequency of magnetic white dwarfs versus effective temperature from all the stars (triangles, same as in Fig. 3) and from nearby white dwarfs (circles).*

Fig. 4 also displays the frequency from all the stars (upper plot from Fig.3). It is seen that the effect of growing fraction of magnetic stars as their temperature drops is essentially potentiated when passing to nearby stars. In the temperature interval 6000–8000 K, magnetic stars account for 16 ± 3.4 %. The fraction of cooler stars (4000–6000 K) equals 13 ± 4 %. The number of magnetic white dwarfs among the oldest and coolest stars possibly decreases, however, this difference in the frequency in these two extreme temperature intervals is not significant. It will be recalled

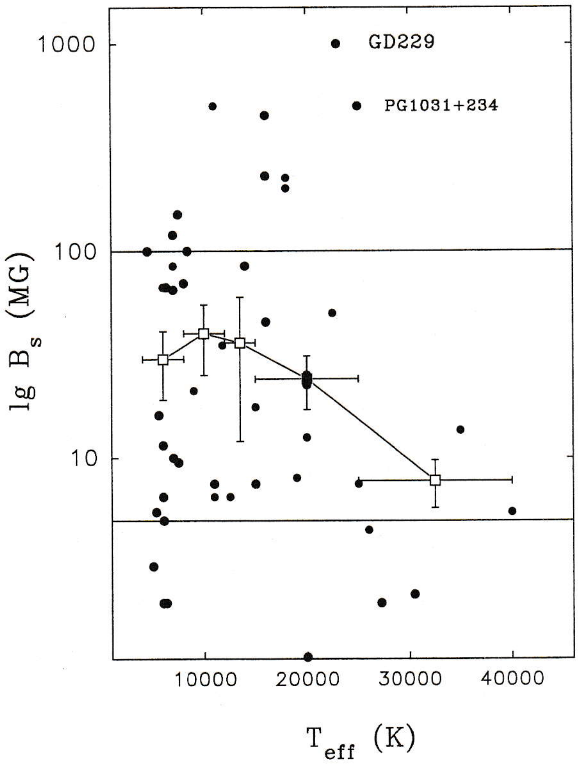

Figure 5: *The relationship between surface magnetic fields and temperature.*

that the temperature intervals we have chosen are wide enough, they are larger than the temperature errors (for a given temperature, see Figs. 1 and 2).

Fig. 5 presents the relationship between surface magnetic fields and temperature. A total of 44 stars with fields above 1 MG are shown on the diagram. One white dwarf with an extremely high temperature, 50000 K (see the table in Fabrika and Valyavin, 1998), is not considered here, although it falls within the general pattern of the distribution (see discussion below). The upper horizontal line corresponds to $B_s = 100$ MG. This is an approximate upper limit of magnetic fields up to which temperatures of magnetic white dwarfs are most reliably determined. The low horizontal line corresponds to $B_s = 5$ MG. This is an approximate value of magnetic field strength below which the probability of discovering a magnetic white dwarf, when visually examining the spectra, depends on line widths, i.e. on temperature. Here the data are strongly dependent on the observational selection. This is illustrated well by the region of avoidance of magnetic white dwarfs in the temperature interval from 8000 K to 25000 K, where the hydrogen lines are very broad.

The plot in Fig. 5 represents the behaviour of the frequency of magnetic stars $P_m(T_{eff})$. Up to magnetic field strengths 150–200 MG the trend towards increasing dispersion and maximum values of the observed surface magnetic fields with decreasing temperature is well visible. The results of averaging of the observational data between the indicated upper and lower magnetic field limits (two horizontal lines), i.e. the temperature — mean surface magnetic field relation, is shown by the boxes. The vertical bars are the scatter of the observed magnetic fields in the specified temperature intervals. This is a monotonic increase of the magnetic field as the stars cool down. With further drop in temperature, $T < 6000$, the mean magnetic field begins to decrease. It is not unlikely that here we observe the process of magnetic field decay. However, to come to a more definite conclusion here, an additional analysis of observational selections is needed.

The functions $B_s(T_{eff})$ and $P_m(T_{eff})$ (Figs. 3, 5) complement each other. The latter illustrates a possible evolution of fields in the most magnetized stars, whereas the former reflects the evolution of magnetic fields in "nonmagnetic" stars. Indeed, as white dwarfs cool down, their magnetic field strengths rise and exceed the critical magnetic field strength $B_s \approx 1$ MG at which the magnetic field becomes observable, so increasing number of stars become "magnetics".

So, both the magnetic fields in white dwarfs and the frequency of magnetic white dwarfs increase as they cool down. A physical mechanism is likely to exist that brings about the increasing of magnetism of white dwarfs with decreasing temperature. The temperature and the age of these stars are directly related through a cooling function. The relationship found can easier be interpreted by considering the frequencies on the scale of ages. Since the cooling of a white dwarf depends strongly on its mass, prior to examination of the frequency of magnetic white dwarfs versus the age, $P_m(t)$, masses of white dwarfs should be estimated.

4. Masses of magnetic white dwarfs, magnetic field — mass relation

Masses of white dwarfs can be estimated on the basis of photometry and parallax data with involvement of the mass–radius relation and the ordinary relationship between the bolometric luminosity, effective temperature and radius (for instance, McMahan, 1989). The most accurate mass estimates can be made on the basis of spectrum modelling. As a rule Hamada and Salpeter (1961) mass–radius relations, derived for a zero temperature and without account of the magnetic field, are used. Results of determination of white dwarfs masses have been published by a number of authors (e.g. Koester et al., 1979; Koester, 1984; Bergeron et al., 1992). Masses of about 200 nonmagnetic white dwarfs have been measured. The present day error of mass determination with the application of the model atmosphere technique is $< 0.1\,M_\odot$ (Bergeron et al., 1992). For bright stars this is from 0.04 to 0.06 $M_\odot$. In order to find how the frequency of magnetic white dwarfs depends on the mass and age, knowledge of masses of as many nonmagnetic stars as possible is required, partly at a sacrifice in mass determination accuracy. That is why below we determine masses of all white dwarfs from the catalogue by McCook and Sion (1999) for which the necessary data are available.

Generally speaking, a consideration of the cooling function with allowance made for the magnetic field are needed. The quantization of orbits of free electrons brings about a decrease in thermal conductivity in the direction normal to that of the magnetic field, but the effect is such that it must be taken into account only in neutron stars, where the magnetic field is as strong as 10^{12} G (Page, 1995). For this reason we assume that the cooling rate does not essentially change because of magnetic fields and use the sequences computed by Salaris et al. (1996) for nonmagnetic oxygen-carbon white dwarfs. The procedure of determining of white dwarf radii is described in sufficient detail by Koester et al. (1979), Shipman (1979), McMahan (1989) and by other authors. The bolometric luminosities can be found from the absolute stellar magnitudes (McCook and Sion, 1999) and from bolometric corrections for white dwarfs of different atmosphere composition. For white dwarfs with strong magnetic fields the bolometric calculations have been lacking yet, therefore we use the corrections given by Bergeron et al. (1995a) computed for nonmagnetic atmospheres of white dwarfs.

From our model estimations the hydrogen lines splitting in the magnetic field have a lesser influence on the bolometric correction than the error in M_v that amounts to 1^m, which corresponds to the mass error of $\approx 0.2 - 0.3 M_\odot$ (see below). It should be noted that the problem of bolometric corrections exists for nonmagnetic white dwarfs as well (Bergeron et al., 1997).

On the basis of the evolutionary sequences of white dwarfs calculated by Wood (1990; 1992 and references therein) it is possible to test the mass–radius relation of Hamada and Salpeter (1961). Using Wood's calculations Bergeron et al. (1992) have found that the inaccuracy in the determination of masses of white dwarfs, when applying Hamada and Salpeter's zero temperature relations, reaches $0.05\,M_\odot$. For the use of Wood's sequences it is necessary to know gravities. It is impossible to derive a gravity directly from photometry data, that is why we further employ Hamada and Salpeter mass–radius relations. Photometric estimates themselves are less accurate than spectroscopic, therefore in the mass–radius relation we can safely use the approximation $T_{\rm eff} = 0$.

Magnetic field is capable of changing the mass–radius relation. At a central field strength of a white dwarf of $10^{12.3}$ G the radius grows by 40 % (Shapiro and Teukolsky, 1985). The maximum observed surface magnetic fields are 10^9 G. The internal magnetic fields in white dwarfs can develop to greater strengths, however until recently the observations provided no evidence in favour of existence of such field inside white dwarfs. Thus, there are no grounds to believe that magnetic fields essentially alter the mass–radius relation, therefore we do not take into account magnetic field.

Thus, we have estimated the masses of 1054 white dwarfs from McCook and Sion (1999), for which the absolute magnitude estimates are given. This list contains 35 magnetic white dwarfs. The bolometric corrections calculated by Bergeron et al. (1995a) are presented separately for the case of purely hydrogen (DA) and hydrogen-weak (spectral classes DB, DO and also DC with temperatures above 6000 K) atmospheres of white dwarfs. Using the data we took into acount the spectral class of white dwarfs (DA or not DA). A small fraction of the stars (less than 5 %) are cold DC with temperatures below 6000 K. There is some uncertainty in choosing a model between hydrogen-rich and hydrogen-weak atmospheres. At these temperatures hydrogen is not noticed in spectra (for instance, Wesemael et al., 1993). The overwhelming majority of the known white dwarfs are stars with hydrogen atmospheres, therefore cold DC white dwarfs are probable to have also a hydrogen atmosphere. The bolometric calculations of hydrogen atmospheres have been used for these stars.

For the white dwarfs of spectral classes DQ (spectra with molecular bands) and DZ (metal-line spectra) the presented bolometric corrections cannot be used because of the absence of calculations of atmospheres with such a chemical composition. For these stars black-body bolometric corrections have been used. This is a crude approximation, but there are no more than 6 % of such stars. Thus, it can be believed that about 90 % of the computed masses are sufficiently reliable.

The shaded histogram in Fig. 6 shows the mass distribution of 1019 nonmagnetic white dwarfs. The absolute magnitudes (McCook and Sion, 1999) were obtained with the use of both trigonometric (if known) and spectral parallaxes. The mean mass in this distribution is $0.65 \pm 0.01 M_\odot$. The unshaded histogram in Fig. 6 shows the masses of 162 DA white dwarfs obtained using the trigonometric parallaxes alone. The mean mass of these stars is $0.57 \pm 0.02 M_\odot$. The latter value is very close to the mean masses of white dwarfs given by other authors. Using the spectroscopic technique, Bergeron et al. (1992) obtained the value of $0.54 \pm 0.01 M_\odot$. Koester et al. (1979) provide an estimate of $0.58 \pm 0.01 M_\odot$ from photometry (with involvement of atmosphere models). Close values are given by a number of other

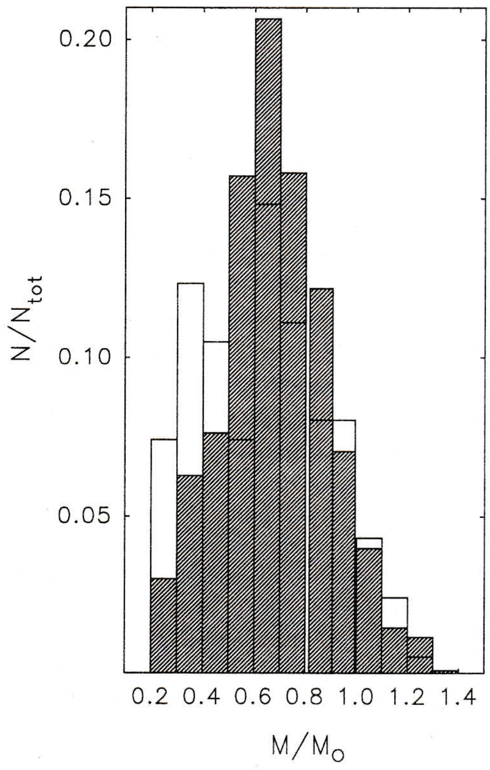

Figure 6: *The mass distribution of nonmagnetic white dwarfs within the different samples.*

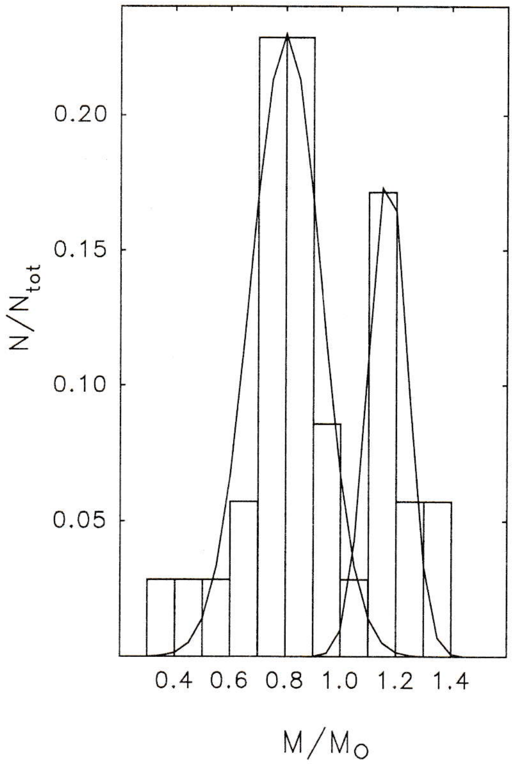

Figure 7: *The mass distribution of magnetic white dwarfs.*

authors (Shipman, 1979; Koester, 1984; McMahan, 1989). Note that in computations of masses the most accurate grids that relate masses, radii and temperatures are used by Bergeron et al. (1992). Other calculations rely on Hamada and Salpeter (1961) relation and show systematically a somewhat higher value for the mean mass of white dwarfs.

The difference in the mean mass estimates obtained from trigonometric parallaxes only and from trigonometric + spectral ones is connected with imperfection of the spectral technique of parallax determination. The published mass estimates of white dwarfs that has been performed by different authors over the last 20 years spread in the mean mass from 0.54 to 0.66 $M_\odot$ (Bragaglia et al., 1995; Bergeron et al., 1995b; 1997). All the results are similar at the level of accuracy 0.1 $M_\odot$. It is seen from Fig. 6 that the determination of absolute magnitudes based on spectral parallaxes introduces on average a systematic error of $+0.08\,M_\odot$. From a comparison of the FWHM of our distribution with that of the distribution given by Bergeron et al. (1992) the estimate of the error of our mass measurement follows, which is approximately equal to 0.15–0.2 $M_\odot$. The mass distribution of non-DA white dwarfs (Ohe et al., 1984) with our accuracy taken into account, is close to that of DA white dwarfs. Thus, loosing by about twice in accuracy of individual estimates, we have a sufficiently large sample of white dwarfs with estimated masses, which is essentially larger than all published.

Masses of magnetic white dwarfs were evaluated in a similar manner. The absolute magnitudes presented in the catalogue by McCook and Sion (1999) were derived both from trigonometric and spectral parallaxes. They used trigonometric parallaxes only for stars showing the values over $0\farcs1$, otherwise parallaxes were estimated from colours of different photometric systems (McCook and Sion, 1987; 1999). For magnetic stars where spectrophotometric parallaxes may involve considerable errors, we used only trigonometric parallaxes, where it was possible. We rid thus the mass estimates of the uncertainty caused by magnetic field. For the star PG 1658+441 (B_s = 2.2 MG) a correct value, M_v = 12.56 was taken from Schmidt et al. (1992).

Fig. 7 displays the mass distribution of 35 magnetic white dwarfs. The average mass value derived from all stars is $0.89 \pm 0.04 M_\odot$. For some magnetic stars (massive, as a rule) from this sample the mass estimates by other authors are available. Among the magnetic white dwarfs, whose masses are available in the literature, our sample has white dwarfs with masses higher than 1.1 $M_\odot$. They are GD 229 ($B_s \sim 1000$ MG), BPM 25114 ($B_s \approx 25$ MG), G 227–035 ($B_s \approx 120$ MG), LP 790–29 ($B_s \approx 150$ MG), G 240–72 ($B_s \approx 150$ MG), PG 1658+440

($B_s \approx 2$ MG), GW+70 8247 ($B_s \approx 230$ MG). Indeed all these white dwarfs have been known to date as "ultramassive" (see Liebert, 1988; Jordan, 1997; Schmidt et al., 1992) with masses greater than 1.1 $M_\odot$. The fact that our mass estimates of the stars (that have received the most study) agree with the data of other authors is evidence of correctness of our estimates of magnetic white dwarf masses.

Liebert (1988) has suggested that magnetic white dwarfs are, on average, more massive than nonmagnetics. Using the diagram $(V - I) + (G - R) - M_v$ Liebert has found magnetic white dwarfs to be less luminous than the same temperature nonmagnetic stars and concluded that they are more massive. Indeed, magnetic white dwarfs are, on average, essentially more massive than nonmagnetics (Figs. 6 and 7). The average masses of these two types of objects differ by $0.24M_\odot$ with an error of $0.01M_\odot$ in normal white dwarfs and $0.04M_\odot$ in magnetics (all stars in Fig. 7).

Besides the difference in the mean masses, we see that the mass distribution of magnetic degenerates is not uniform. Analysis has shown that it differs from normal distribution with a confidence level of > 90%. The distribution in Fig. 7 in the interval of masses over $1.1M_\odot$ differs from that given in Fig. 6. From the distribution in Fig. 7 one can see the existence of two different populations of magnetics — a "massive" population with masses greater than about 1.1 and a "normal" one having masses smaller than $1.1M_\odot$. The solid lines represent the Gaussian approximations of the two populations. They provide the values of the distribution maxima $M_c = (0.8 \pm 0.03)M_\odot$ for the latter population and $M_c = 1.15M_\odot$ for the former. The widths of these distributions FWHM $= 0.2M_\odot$ and FWHM $= 0.1M_\odot$ for both populations, respectively.

Now we examine relative frequencies of magnetic and nonmagnetic stars of masses $< 1.1M_\odot$ and $> 1.1M_\odot$ The frequency of magnetic white dwarfs in the region of masses $< 1.1M_\odot$ (with an accuracy that takes account of only the random statistics) is 0.03 ± 0.01. This value in the region $> 1.1M_\odot$ is $0.21 \pm 0.02M_\odot$. The frequency of magnetics among very massive stars is by a factor of 7 higher than among white dwarfs of ordinary masses. Let us consider the relative numbers of massive stars in Figs. 6 and 7. Among the nonmagnetic white dwarfs the relative number of stars with masses $> 1.1M_\odot$ is 0.03, whereas their number amounts to 0.29 among magnetic degenerates. It is seen that massive stars among magnetics are by a factor of 9 more numerous than among nonmagnetics.

Ten stars out of 35 magnetics presented in Fig. 7 have masses $> 1.1M_\odot$. It is very important that 7 stars out of these 10 stars have magnetic fields $B_s > 100$ MG. Among the remaining 25 magnetic stars of "ordinary" masses there is not a single star with a magnetic field exceeding this value. Thus, the frequency of strongly magnetic ($B_s > 100$ MG) stars among magnetic white dwarfs of ordinary masses equals zero, while among very massive ones it makes 70%.

From examination of Fig. 7 one can assume that among the 10 "ultramassive" magnetic white dwarfs with masses exceeding $1.1M_\odot$ three stars, in fact, belong to the ordinary population, in which no stars with fields > 100 MG are observed. These three stars and also 3 least massive ($M < 0.6M_\odot$) stars may well represent the wings of the symmetric distribution of magnetic white dwarfs of "ordinary" masses — the main population of these stars. The remaining 7 most massive and most magnetics represent the second population of magnetic white dwarfs. The proportion of the stars that belong to the second population of magnetic white dwarfs ("ultramassive"–"ultramagnetic") is $\approx 7/35 = 0.2$.

In the above discussion it was indicated that strong magnetic fields ($B_s \geq 100$ MG) produce the blanketing effect essential for hot white dwarfs: the depression in UV and the brightness increase in visible. It may well be that the blanketing does not distort strongly the B–V index, i.e. has no influence on the temperature measurements. However the temperature of majority of the most magnetized white dwarfs was determined by different authors using the model atmosphere method with allowance made for the magnetic field. Because of the blanketing one may overestimate visual brightness of magnetic degenerate (M_v), i.e. overestimate the radius and underestimate the mass. Using the nonmagnetic bolometric corrections results in overestimating luminosities too. So the masses of the ultramagnetic white dwarfs may be underestimated. Taking into account these effects have to strengthen the observed separation of two populations of magnetic white dwarfs in masses.

So, two independent conclusion follow from analysis of masses of white dwarfs (Fig. 6 and 7).

1. Magnetic white dwarfs are, on the average, more massive than nonmagnetics.

2. The sample of magnetic white dwarfs is not homogeneous. It consist of two types of objects: "ordinary" mass white dwarfs whose magnetic field $B_s < 100$ MG and massive (to be more precise, "ultramassive") white dwarfs, whose magnetic field $B_s > 100$ MG.

Let us refine the mean masses of magnetic white dwarfs. The mass of magnetic degenerate is larger than that of a nonmagnetic by 0.24 ± 0.04 $M_\odot$. If the two populations of magnetics are examined separately, the mean mass of the main magnetic population exceeds then the mean mass of a nonmagnetic star by 0.15 ± 0.03 $M_\odot$, while a white dwarf from the "ultramassive–ultramagnetic" population is, on the average, by 0.4 $M_\odot$ more massive than a non-magnetic white dwarf.

The idea that magnetic white dwarfs represent two different populations: "normal" stars and ultra-massive has already been discussed in the literature (Jordan, 1997). Probably the two types of objects have different history of evolution. For instance, the stars of the latter population have passed evolution in binaries, or even a particular evolutionary stage of merging two white dwarfs, that is why these are massive single stars. In any case observations of magnetic white dwarfs for variability of radial velocities and examination of their spectra in near IR for a search for secondary star seem to hold much promise.

5. Frequency of magnetic white dwarfs versus a lifetime

Our sample of white dwarfs for which effective temperatures can be estimated in a unified fashion contains 1274 stars. It includes 38 magnetic stars, and their share in the total sample is 0.03. We have isolated stars for which we can provide mass estimates. This subsample comprises 1054 white dwarfs 35 of which are magnetics. The share of magnetics is also 0.03. The equality of these two ratios suggests that the subsample of magnetic white dwarfs is unbiased from the point of view of determination of temperatures and masses and, therefore, ages as well.

In age estimation of white dwarfs we use the sequences of Salaris et al. (1996) for nonmagnetic oxygen-carbon white dwarfs. The sequences are given for masses 0.54–1.0 $M_\odot$. We have interpolated these sequences to a value of 1.2 $M_\odot$. In determining ages we used the unified evolutionary sequences $t = f(T_{eff}, M)$ for both nonmagnetic and magnetic stars.

The frequency of magnetic white dwarfs having fields over 1 MG is presented in Fig. 8. Filled circles indicate the frequency obtained for all magnetic white dwarfs. Open circles are for magnetic white dwarfs in the vicinity of the Sun, d<25 pc. The horizontal bars show the age intervals inside which the frequency was averaged; the vertical ones are statistical errors. As it is seen from the figure the frequency of magnetic degenerates shows regular tendency to increase with age. The value of P_m increases even despite the fact that with increasing age (with cooling and reduction of brightness) of white dwarfs, detection of their magnetic fields gets more difficult. Such a selection of magnetic stars with growing age has, in principle, to diminish P_m.

The rise of $P_m(t)$ is regular, the maximum and minimum values differ at a confidence level of 0.9995. The relation constructed from the sample of nearby stars still further strengthens the observed effect.

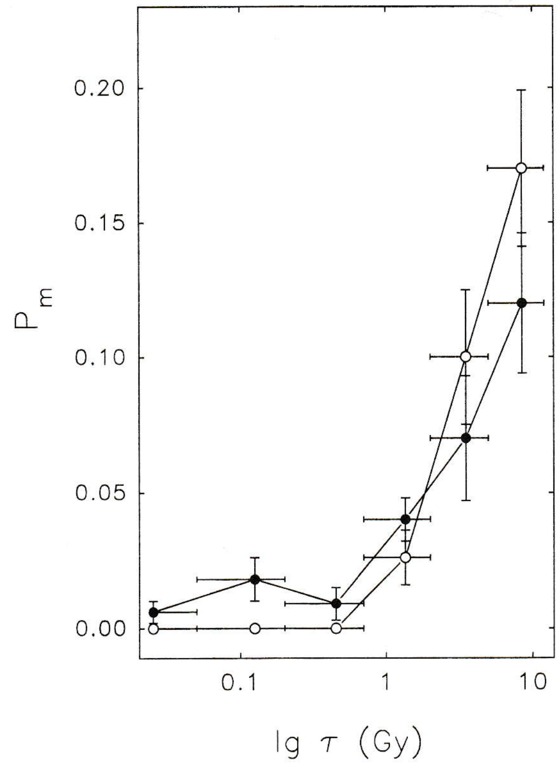

Figure 8: *The frequency of magnetic white dwarfs depending on their age.*

Considering only nearest stars we take into account the strongest observational selection: the cooler the white dwarf, the fainter it is and the more difficult to find such stars and, moreover, to recognize their magnetic nature. So, we believe that the increase in the frequences of cool magnetic degenerates is real.

Thus we have found magnetic fields in white dwarfs to grow as the stars cool down. Obviously this is the cause of increasing frequency of magnetic white dwarfs with age. The diagrams of frequencies of magnetic white dwarfs versus temperature and age (Fig. 4 and 8) and the diagrams magnetic field — temperature and age (Fig. 5 and 9) are independent and complement each other. Indeed, the frequency behaviour suggests that with decreasing temperatures an increasing number of stars go from the region of nonmagnetics ($B_s \leq 1$ MG) to the region of magnetics, i.e. new magnetic white dwarfs appear in the course of the evolution. This itself does not mean that on the diagrams $B_s(T_{eff})$ or $B_s(t)$ we have to see any relationships. It can, however be argued that if the function of magnetic field evolution $B(B_0, t)$ is independent or only slightly dependent on the initial magnetic field B_0, we will see relationships on diagrams $B_s(T_{eff})$ and $B_s(t)$. These diagrams suggest that the evolution function may really be independent on the initial magnetic field. The same conclusion follows from analysis of current and initial mag-

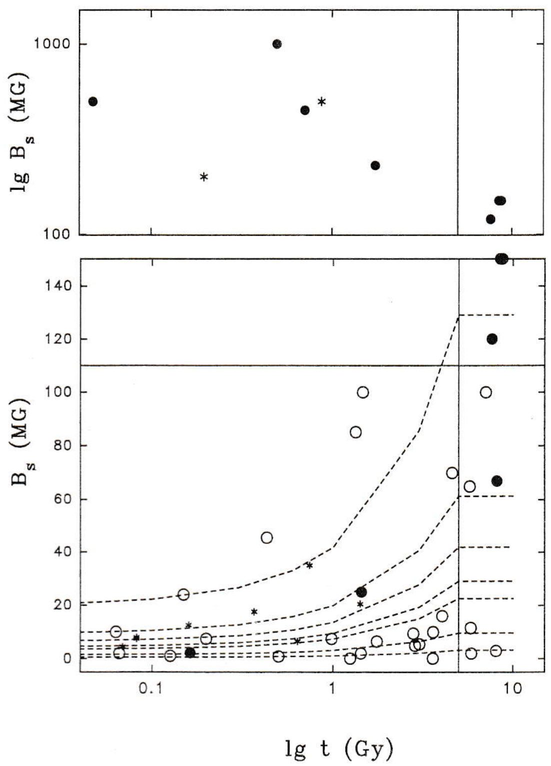

Figure 9: *The relationship between surface magnetic fields of white dwarfs and their ages.*

netic field functions (Fabrika and Valyavin, 1998). So we can claim here from the diagrams $P_m(T_{eff}, t)$ that magnetic field evolution does exist, and assume from the diagrams $B_s(T_{eff})$, that at least at field strengths $B_s \leq 100$ MG the evolution is likely to be independent on the initial magnetic field.

It is seen from Fig. 5 that beginning with temperatures of about 10000 K, the magnetic fields cease gradually to rise, further they show a tendency to decay. Such a surface temperature corresponds approximatively to beginning of crystallization in a white dwarf of 0.6 $M_\odot$ (Lamb and Van Horn, 1975; Bisnovaty–Kogan, 1989). This may suggest that magnetic field intensification mechanism is discontinued by crystallization and we further-may observe a decay of magnetic fields. The decay time is about 10 Gy, what is longer than cooling time (Wendell et al., 1987). The observational data available now do not allow the field decay to be studied.

Fig. 9 shows the diagram surface magnetic field — age. This diagram includes 48 stars. Different symbols indicate the known magnetic white dwarfs having age estimated. Filled circles mark massive stars of masses over 1.1 $M_\odot$. Open circles are for stars of smaller mass. Asterisks show white dwarfs whose masses are unknown, $M = 0.8 M_\odot$ was accepted for them. As it can be seen from Fig. 7, this is an approximate average mass of the main population of magnetic white dwarfs. In magnetic field strengths the diagram is divided into two parts: the lower part up to 150 MG and the upper one on a logarithmic scale over 100 MG. Up to 150 MG the observed magnetic fields of white dwarfs clearly tend to increase with age. At higher field strengths we see the separate population of ultramassive–ultramagnetic stars. Fig. 9 confirms the assumption stated above that ultramassive and ultramagnetic white dwarfs represent an isolated class of objects.

The problems of origin and evolution of magnetic fields in white dwarfs were studied by many authors. Wendell et al. (1987) presented calculations of an axial-symmetric magnetic field decay in white dwarfs. The magnetic field is believed to be residual from the field of a precursor star. The presence of residual strong magnetic fields in white dwarfs is explained by the existence of the main sequence magnetic stars — precursors of magnetic white dwarfs (Angel et al., 1981; Schmidt and Smith, 1995; Putney, 1997). Another approach to the problem suggests consideration of dynamo mechanisms (for instance Markiel et al., 1994; Thomas et al., 1995). Our results show that in any case one should examine both points of view, especially at early stages of the white dwarfs evolution. From our data it follows that, beginning at least with the age of 10^7 years (the temperature is about 30000 K), the surface magnetic fields of white dwarfs are sure to rise. We will discuss two possible interpretations of the effect:

- increasing electric conductivity in a white dwarf assuming a constant electromotive force;
- outward diffusion of the magnetic field "buried" at the star formation.

The latter scenario has been proposed to us by A.I. Tsygan.

In Fig. 9 three massive white dwarfs with a relatively weak magnetic field (<100 MG, as we assumed when examining Fig. 7) can well represent a "massive tail" of the main population of magnetic white dwarfs. They follow well the general trend for field strength increase as white dwarfs of "ordinary" masses are cooled down.

We see that 7 stars of ultramassive ($> 1.1 M_\odot$) and ultramagnetic (>100 MG) population in the upper part of Fig. 9 stand out also for the behaviour of $B_s(t)$. Indeed, an apparent decrease in the magnetic field strength with age is observed there. The time of magnetic field two–fold decay in these objects is observed to be about 3 Gy. Calculations of magnetic field decay time in white dwarfs yield a value of about 20 Gy for the main dipole eigenmodes (Wendell et al., 1987). With growing mass (down in radius) this time decreases as $\propto R^2$. In particular, the radius of a white dwarf of $M > 1.15 M_\odot$ is < 0.5 of a radius of white dwarf with $M = 0.6 M_\odot$. Following Wendell et al. (1987) one can find that for a white dwarf mass $M > 1.15M_\odot$ the decay time of a dipole field is < 5 Gy that agrees well with the value obtained from Fig. 9. The decay time of magnetic field of a more complex geometry (quadrupole, etc.) is shorter.

Thus, we may suppose that in the case of "ultramassive–ultramagnetic" degenerates we observe directly the process of magnetic field decay, and the decay rate being close to that given by theory.

The former scenario of magnetic field rise is based on the assumption of existence of an electromotive force in these stars. The electric conductivity in white dwarfs rises with star cooling. Pikel'ner (1966), Shlyuter and Birman (1954), Shlyuter (1956a, b) and other authors discussed different scenaria of the e.m.f. generation of closed currents in a star. Those currents produce magnetic fields.

The electric conductivity rise with time in the presence of the external e.m.f. can account for the observed increase in magnetic fields. If the e.m.f. is assumed constant and for some reason kept until crystallization is over, one is likely to observe an increase in the magnetic field strength as a white dwarf cools down. In this scenario the magnetic field strength in a degenerate will change with time in proportion to the variation of its electric conductivity. The conductivity in white dwarfs has been calculated. We use here the computations of Wendell et al. (1987). The dashed lines in the lower panel of Fig. 9 show the magnetic field evolution in this scenario. The e.m.f. is believed to exist there up to the point of complete crystallization, that is about 5 Gy for a white dwarf of $0.6M_\odot$. In spite of the rough assumption taken in this scenario, the general agreement between the conductivity tracks and the observation in Fig. 9 suggests that this scenario calls for a more serious examination.

The second interpretation of the degenerate magnetism rise with time suggests the magnetic field to diffuse from inner region outward. At a white dwarf formation during the red giant core collapse because of neutrino cooling the interior contracts faster than the outer parts (Wendell et al., 1987). Magnetic fields are "sucked in", and the gas accreted from the outer parts covers magnetic fields. Wendell et al. (1987) performed their computations disregarding rotation, however, they found that even so the geometry of the initial magnetic field may be essentially distorted. The buried magnetic field diffuses outward for a time scale $t \approx 4\pi\sigma R^2/c^2$ (Pikel'ner, 1966), where R is the characteristic layer size the magnetic field diffuses across, σ is the electric conductivity. Magnetic field diffusion and decay are in fact the same processes. When diffusing in the conductor the field is decaying at about the same time because of ohmic losses.

From our data (Fig. 9, bottom) one can roughly estimate that the time of e–fold increase in magnetic field strength in a white dwarf of ordinary masses is 2.5 Gy. If this time is that of diffusion, then we can find the depth of field original location inside a white dwarf. It follows from Wendell et al. (1987) results that the electric conductivity at degenerate center right after its formation is about $10^{21}s^{-1}$ and 10 Gy later it is $\approx 3 \cdot 10^{22}s^{-1}$. Taking $\sigma = 10^{21}s^{-1}$ find the depth of location to be $R \approx 7.7 \cdot 10^7$ cm, which is only 10 % of the radius of a white dwarf of $0.6M_\odot$. Considering the fact that the conductivity in the outer degenerate parts is about one order of magnitude lower, then the depth of field location immediately after the white dwarf formation will turn out to be 30 % of its radius. We see that our estimates lead to quite real values, i.e. they agree with the idea of diffusion of the buried magnetic field.

Ultramassive–ultramagnetic degenerates do not show the magnetic field strength to increase with time as it is the case with magnetic stars of ordinary masses. Ultramassive white dwarfs are likely to have passed evolution as binary systems. There are two possible ways here.

1. The ultramassive–ultramagnetic degenerates were formed as a result of merging of two white dwarfs. Their magnetic fields might then turn out to be at the surface right after the formation of a new degenerate, that is why only magnetic field decay is observed.
2. These stars were formed in cataclysmic variables, where a white dwarf becomes a more massive because of a standard gas accretion. Then the temperature of the accreting (massive) white dwarf rose as a result of accretion. Because of accretion heating the age of such stars is actually greater than that we have obtained by a standard cooling function of white dwarfs. For this reason their magnetic fields have already diffused outward, and we therefore observe only the effect of the field decay.

In this paper we do not aim at interpreting the found effects of magnetic field evolution in white dwarfs. Nevertheless, we have discussed two probable scenaria of enhancement of magnetism of white dwarfs with time and shown both of them to be consistent with observations. We have also suggested two interpretations of the found population of "ultramassive–ultramagnetic" degenerates. This population itself carries a great interest. It is extremely important to study in detail rotation periods and magnetic field geometry of ultramassive-ultramagnetic white dwarfs. Such data can give a light in understanding of these stars.

6. Conclusion

Based on photometry and parallax data, we have estimated temperatures of 1225 nonmagnetic and 49 magnetic white dwarfs and masses of 1019 nonmagnetics and 95 magnetics. Accuracies of these parameters have been discussed. We had the task also to analyze systematic errors in determination of temperatures and masses of white dwarfs. Our temperature and mass estimates are less accurate than the estimates made using the model atmosphere methods. However, we have obtained in this way a considerable sample of white dwarfs, quite sufficient for statistical studies.

The frequency of magnetic white dwarfs has been found to be strongly dependent on temperatures of these stars and, therefore, on their age. We have thus confirmed the suggestion of Liebert and Sion (1979) that magnetic white dwarfs are more frequent among cool stars than among hot stars. Possible effects of observational selection are discussed and taken into account. Taking account of observational selections leads to an essential increase in the amplitude of the found magnetic field evolution. With decreasing temperature of degenerates, the number of magnetic stars among them and also the average magnetic field strength of a white dwarf grow with a typical time of $t \approx 2-3$ Gy and reach a maximum at a photosphere temperature of about 6000–8000 K. With further cooling the mean magnetic field is observed to decrease, which is possible due to its decay. However this effect is not significant. It has been assumed that the magnetic field evolution function of degenerates $B(B_0, t)$ is independent (or slightly dependent) on the initial magnetic field B_0.

The observed evolution of magnetic fields is discussed within the frames of two scenaria:

- the change in magnetism represents the evolution of electric coductivity inside the star under the condition of constant external e.m.f;
- the magnetic field "buried" during formation of a degenerate diffuses outward.

A comparative analysis has been made of masses of magnetic and nonmagnetic white dwarfs. Magnetic degenerates have been found to be more massive. This is consistent with Liebert's (1988) result. The mean mass of a magnetic white dwarf is about $0.25M_\odot$ larger than the average mass of a nonmagnetic degenerate. The frequency of magnetic stars among the most massive degenerates is nearly an order of magnitude higher than among white dwarfs of normal mass.

It has been found that magnetic white dwarfs themselves are not a homogeneous class of objects. They consists of two populations: degenerates of "ordinary" masses whose magnetic fields $B_s < 100$ MG; "ultramassive–ultramagnetic" white dwarfs with masses $\geq 1.1M_\odot$ and magnetic fields $B_s > 100$ MG. Among the stars of the second population the magnetic field decay with a time scale of ≈ 3 Gy is observed. Such stars account for about 20 % among magnetic white dwarfs. We believe that magnetic white dwarfs of this population have evolved in binary systems. They may be resulted from merging of two degenerate stars or they may accumulate mass as a result of gas accretion in standard CV scenaria. Thus if the stars of this type were formed as a result of merger, then they are single stars; if there were no coalescence, then the second component have to be a low-mass degenerate or a planet. In the second case the component may be markedly cooler than the observed magnetic white dwarf because of the cooling time of a white dwarf depends strongly on mass.

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