# Gamma-ray bursts as an instrument for testing cosmological models

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**Abstract** We present a review of possible cosmological applications of Gamma-Ray-Bursts multi-wavelength observations. A statistical analysis of the main pulse parameters of BeppoSAX, BATSE, Fermi and Swift GRB observational data is conducted. The spatial distribution of GRB sources with known redshifts is analyzed. Selection effects that distort the true source distribution are taken into account by comparing the observed distribution with fractal and uniform model catalogs. We review GRBs as standard candles and test of Universe isotropy by GRBs. We approximated trend of cosmological time dilation by long GRBs and discuss in detail possible effects that influence their statistics.

**Key words:** Gamma-Ray Bursts, Cosmological Models, Hubble's Diagram, Time Dilation, Large Scale Structure, Fractal Dimensionality

## 1. Introduction

Cosmology as a part of physics is based on the experiments/observations, which are used for testing the validity of existing theoretical models of the Universe. Edwin Hubble (1937) in his book *The Observational Approach to Cosmology* [13] defined it as the science which study the fair sample of the Universe, i.e. the observable region of space that can be explored with existing instruments. Sandage (1995) [27] gave the name "**Practical Cosmology**" to such strategy for the large Universe exploration and formulated 23 basic unsolved yet astronomical problems (on the analogy with "23 Gilbert problems" for mathematics). His list includes also the nature of the cosmological redshift (the 15<sup>th</sup> Sandage's astronomical problem). This observational approach to cosmology was further developed by Baryshev & Teerikorpi (2012) [4] (on the problem of cosmological redshift see [3]).

Modern state of the practical cosmology includes the multimessenger astronomy which unites observations of electromagnetic radiation (from radio up to gamma bands), cosmic rays, neutrino and gravitational waves. These new observational situation opens new possibilities for testing the basis of the cosmological models by using the observational approach of the practical cosmology. Last review of the luminosity correlations of GRBs, and implications for constraining the cosmological parameters and dark energy was given by Wang F.Y. et al. (2015) [34]. Further research of the GRBs afterglows, generally having broken power-law spectra, will give possibility to extract intergalactic medium (IGM) absorption features.

Here we give a review of cosmological tests which use observations of gamma-ray bursts (GRB) especially in view of the forthcoming mission Transient High Energy Sky and Early Universe Surveyor (THESEUS) (Amati et al. (2018) [1]; Strata G. et al. (2018) [33]). Preceding review of the GRB cosmology was given by Petrosian et al. (2009) [22].

In Sec.2 we discuss a classification of cosmological tests and formulate those observational relations which will be used in our review. Sec.3 is devoted to data analysis of the main GRB missions. In Sec.4 we discuss main results from the analysis of spatial distribution of GRB sources, test of Universe isotropy by GRBs, the Hubble diagram for GRBs and consider the cosmological time dilation effect. Sec.5 presents general discussion of  $T_{90}$  trends. Conclusion is given in Sec.6.

# 2. Classification of cosmological tests

In the framework of practical cosmology it is important to distinguish between *theoretically inferred cosmological laws* and *empirically measured cosmological relations*, which are used for testing theoretical models (Baryshev & Teerikorpi [4]). There are several possible classifications of observational cosmological tests, which include *parametric* tests (such as derivation of main parameters of the standard cosmological model) and *crucial* tests (such as establishing the nature of the large-scale structure of the Universe and the nature of the cosmological redshift). According to Orlov & Raikov [19] there is also general division of cosmological tests on two classes – *cosmographic* and *physical* tests.

Modern practical cosmology includes the following observational tests of world models:

- testing the validity of general relativity at largest scales;
- testing the nature of cosmological redshift and reality of space expansion;
- correlation properties of the matter distribution on largest scales;
- measuring the temperature of the background radiation at different redshifts;
- determining the ages and chemical composition of the high redshift objects in the observable Universe.

The main obstacle in extracting true physical information from the directly observed empirical relations is the different kinds of selection and distortion observational technical and physical effects, e.g. K-correction, absorption, evolution, Malmquist bias and others [4]. The most difficult part of cosmological tests is related to careful taking into account such hidden distortion effects.

Modern cosmology is developing also *neo-classical* tests which include very high redshift observations of new type objects and phenomena, such as gamma-ray bursts, very high energy cosmic rays, neutrino and gravitational radiation. For our review we choose following cosmological tests:

- The spatial distribution of GRB sources
- Hubble diagram for GRB sources
- Testing isotropy and inhomogeneity of the Universe by GRBs
- Cosmological time delay in GRB light curves

These tests relate to the fundamental aspects of cosmological models.

# 3. GRB missions

## 3.1. The BATSE

The Burst and Transient Source Experiment (BATSE) by NASA's Marshall Space Flight Center searched the sky for gamma-ray bursts (in range from 20 to >600 keV) and conducted full sky surveys for long-lived sources. The BATSE is an instrument on the Compton Gamma Ray Observatory (CGRO) in the Earth orbit which detects photons with energies from 20 keV to 30 GeV. The observatory was launched from Space Shuttle Atlantis on April 5, 1991, and operated until its deorbit on June 4, 2000. Successors to CGRO include the ESA INTEGRAL spacecraft (launched 2002), NASA's Swift Gamma-Ray Burst Mission (launched 2004) and NASA's Fermi Gamma-ray Space Telescope (launched 2008). Description of the Fourth BATSE Gamma Ray Burst Catalog was done by Paciesas et al. in [20].

#### 3.2. The BeppoSAX

The Gamma-Ray Burst Monitor aboard the BeppoSAX satellite was an Italian–Dutch satellite for X-ray astronomy which played a crucial role in resolving the origin of gamma-ray bursts (GRBs). It was launched on 30 April 1996 into a low inclination (<4 degree) low-Earth orbit. The expected operating life of two years was extended to April 30, 2002 due to high scientific interest in the mission and the continued good technical status. BeppoSAX was named in honor of the Italian physicist Giuseppe "Beppo" Occhialini. SAX stands for "Satellite per Astronomia a raggi X" or "Satellite for X-ray Astronomy". It was the first X-ray mission capable of simultaneously observing targets over more than 3 decades of energy, from 0.1 to 300 (keV) with relatively large area, good (for the time) energy resolution and imaging capabilities (with a spatial resolution of 1 arc minute between 0.1 and 10 keV). The BeppoSAX catalog is represent by Frontera et al. in [8].

## 3.3. The Swift

The Neil Gehrels Swift Observatory [10], previously called the Swift Gamma-Ray Burst Mission, is a NASA space telescope designed to detect gamma-ray bursts (GRBs). It was launched on November 20, 2004. The observatory has the following instruments: Burst Alert Telescope, X-ray Telescope and Ultraviolet/Optical Telescope. Burst Alert Telescope (BAT) detects GRB events and computes their coordinates in the sky. It locates the position of each event with an accuracy of 1 to 4 arc-minutes within 15 seconds. Energy range: 15–150 keV. X-ray Telescope (XRT) can take images and perform spectral analysis of the GRB afterglow. This provides more precise location of the GRB, with a typical error of approximately 2 arcseconds radius. Energy range: 0.2–10 keV. Ultraviolet/Optical Telescope (UVOT). After Swift has slewed towards a GRB, the UVOT is used to detect an optical afterglow. The UVOT provides a sub-arcsecond position and provides optical and ultra-violet photometry through lenticular filters and low resolution spectra (170–650 nm) through the use of its optical and UV prisms.

#### 3.4. The Fermi

The Fermi Gamma-ray Space Telescope (FGST), formerly called the Gamma-ray Large Area Space Telescope (GLAST), is a space observatory being used to perform gamma-ray astronomy observations from low Earth orbit. Fermi was launched on 11 June 2008. Its main instrument is the Large Area Telescope (LAT), with which astronomers mostly intend to perform an all-sky survey studying astrophysical and cosmological phenomena such as active galactic nuclei, pulsars, other high-energy sources and dark matter. Energy ranges from 20 MeV to 300 GeV. Another instrument aboard Fermi, the Gamma-ray Burst Monitor (GBM) (formerly GLAST Burst Monitor) detects sudden flares of gamma-rays produced by GRB and solar flares. Its scintillators are on the sides of the spacecraft to view all of the sky which is not blocked by the Earth. The design is optimized for good resolution in time and photon energy. Ranges: from 8 keV to 1 MeV and from 150 keV to 30 MeV. Review of The Fermi GBM Burst

Catalog by Narayana Bhat P. et al. is available in [17].

## 4. Cosmological tests

### 4.1. Spatial distribution of GRB sources

One of the fundamental requirements in the  $\Lambda$ CDM model is homogeneity of the matter distribution on a large scale. However recent works in this area [12,15] reveal more and more inhomogeneity up to ~600 Mpc. The new evidences of existence of superstructures with sizes about 1000 Mpc in beam surveys are given in [28, 2]. These facts require elaboration of the cosmological model parameters by investigating visible matter spatial distribution, for example, fractal analysis [4]. Since GRBs are indicators of galaxy clusters [31, 32] it should be expected a correlation between statistical properties of galaxy spatial distribution and GRBs. So a statistical analysis of minimal distances between GRBs indicates a galaxy cluster, e.g. five GRBs with coordinates  $23^{h}50^{m} < \alpha < 0^{h}50^{m}$ ,  $5 < \beta < 25$  and the redshift of 0.81 < z < 0.97 [9,29,30].

As is emphasized in [4, Ch.10, 11] the Peebles's correlation function [21]  $\xi(\mathbf{r})$  is strongly distorted by the borders of real samples and to get robust statistical characteristics of the spatial distribution of galaxies one should use the conditional density function  $\Gamma(\mathbf{r})$ . In particular for a fractal spatial distribution the slope of power-law  $\Gamma(\mathbf{r})$  gives the robust estimation of the fractal dimension D (for homogeneous galaxy distribution D = 3).

In papers [9,23] the fractal dimension of GRBs spatial distribution was estimated in the interval D = 2.2 to D = 2.7. New method of fractal analysis proposed in Raikov & Orlov [24] and developed by Shirokov et al. [29,30] allows to estimate fractal dimension on all sample scales where statistics of pairwise distances is defined. In Figure 1 we present the normalized distributions of the pairwise distances for real GRBs sample and for fractal model catalogs with D = 2.0 and D = 2.5. The horizontal line corresponds to uniform Poison's law of spatial points distribution.



Fig2. The distribution of pairwise distances (Mpc) for the Swift GRBs sample and model fractal catalogs.

#### 4.2. Isotropy and inhomogeneity of the GRB Universe

Isotropy of the GRBs distribution over the celestial sphere by the Fermi, BATSE and Swift data was analyzed in paper [25]. Authors considered the observed properties of GRBs and made the conclusion: "...the results are consistent with isotropy confirming".

It must be emphasized that homogeneity and isotropy of spatial distribution are different properties of the large-scale structure. For example the fractal matter distribution can have statistical isotropy and simultaneously be strongly inhomogeneous. Our above results in Sec.4.1 about discovery of fractal dimension D close to 2 on very large scales demonstrate that such situation may be realized in the GRB spatial distribution.

In the framework of the fractal cosmological model suggested in Baryshev [3] the Universe is isotropic and inhomogeneous having fractal dimension D = 2.0. Such value of the fractal dimension guaranties the linear redshift-distance law, if the cosmological redshift is interpreted as the global gravitational redshift within fractal structure on scales where such structure exists.

#### 4.3. Hubble diagram for GRB sources

Measurements of the Hubble diagram of Type Ia supernovae (SNIa) provided the first direct evidence for cosmic acceleration [18]. However we can construct the Hubble diagram using SNIa up to z < 1.5 only. GRBs provide opportunity to continue the magnitude - distance dependence by a sample of GRBs with z > 1.5 (presented in Figure 2). It should be noted that the error bars of SNIa are about 1-2 orders smaller than the luminosity determined errors of GRBs. The behavior of the best fitted curve (red) is close to the standard  $\Lambda$ CDM model (grey) with matter density parameters  $\Omega_m$ = 0.24 and  $\Omega_m$ = 0.33 respectively. An agreement with the  $\Lambda$ CDM model also has been obtained in [35]. Future THESEUS observations will give essential extension and accuracy of the observed Hubble law.



Fig2. The Hubble diagram for SNIa (blue points) and GRBs (black points) with ACDM prediction (grey curve) and approximation (red curve) from Demianski M. (2017) [7].

#### 4.4. GRB time dilation and the nature of cosmological redshift

One of the simplest crucial cosmological tests on the nature of cosmological redshift is the measurement of duration of known physical processes in high redshift objects (cosmological time dilations). Studies of the SNIa light curves made by Goldhaber et. al.(2001)[11] and Blondin et al.(2008)[5] leads to conclusion about the (1 + z)-law.

It is important to test the time dilation effect also for GRB phenomenon. We divide the Swift GRBs in long ( $T_{90} > 2s$ ) and short ( $T_{90} < 2s$ ) ones that are less than 10% in the sample and approximate the trend by least squares for middle points, separately for long and short GRBs in Figure 3. There is no trend up to z = 3. At 3 < z < 4 there is a significant congestion of GRBs in the region of 100 seconds, although this is not observed at z > 5.



*Fig3.* Distribution of  $T_{90}$  verse redshift in log-scale separated by 2-second line.

## 5. Discussion

#### 5.1 About T<sub>90</sub> trends

The  $\Lambda$ CDM model prediction is  $T_{90} \sim (1+z)$  for all time events at high redshifts in the Universe. The observational data analysis (without taking into account hidden selection effects) shows no trend for the separated sample with durations larger than 2 seconds. Therefore, either no time dilation effect or separating short GRBs (at the 2 second level) is incorrect. To verify the first statement it is necessary to discuss observational selection effects, e.g. D. Kocevski & V. Petrosian (2013) [14]. We will analyze these effects later. In this work we suggest to pay attention to principal difference between short GRBs and long GRBs. That is, we are talking about a new effect of observational selection, namely an incompleteness effect of GRBs sample.

The long GRBs are explosions of supernovas, but the short GRBs are merges of binary

systems. These events have a different nature and different light curves Sokolov et al. [32]. It may be that our instruments do not observe short GRBs at high redshifts, thus it is necessary to separate these two subsamples of GRBs. Since short GRBs are less than 10% in the sample, it is possible to use the rough constraint of 2 seconds. Even if a part of long GRBs is shorter than 2 seconds, this effect is insignificant due to a little amount of points, which is shown in Figure 4d for approximation from 1 up to 4 of redshift.

We also considered alternative approximations and compared them with other papers. The averaging Swift data in Figures 4a and 4b give a trend (1+z) for approximation up to z = 8, but with a large margin of error  $(\pm 0.5)$ , a similar results were obtained in [36]. A  $T_{90}$  trend at average points of all Swift GRBs is  $(1+z)^{1.5\pm0.2}$  at z < 4 and this result is most reliable and Figure 4c shows it. A similar result  $T\sim(1+z)^{1.4\pm0.3}$  has been obtained for the radio loud GRBs sample in paper [16].



Fig4. The different approximations of  $T_{90}$ .

Duration of GRBs strongly depends on spectral sensitivity and other instrument features [6,14]. For example, GRB 150101B in the Fermi/GBM catalog has  $T_{90} = 0.018$  sec., and at the same time its  $T_{90} = 0.080$  sec in the Swift catalog. Also it is necessary to verify a light curve expansion (the GRB time dilation essence) with redshift rate by analogy with supernovae [5,11]. However these findings could be not secure in view of Orlov & Raikov [19]. Recently it was shown that some otherwise "short" GRBs have  $T_{90}$  durations up to several minutes: these events are only short in the literal sense, e.g. Rueda et al [26]. This and similar facts also demonstrate that the definition of long and short GRBs should be reconsidered.

## 6. Conclusion

Future THESEUS space observations of GRBs [1, 33] and corresponding multimessenger ground-based studies including large optical telescopes [6, 31, 32] will bring crucial

information for testing theoretical cosmological models.

Our summary:

- The fractal dimension estimation of the Swift GRB spatial distribution gives  $D = 2.55 \pm 0.06$  on scales of 1.5 5.5 Gpc. This is an indirect indication that the Universe is inhomogeneous on these scales.
- The Universe is isotropic based on the BATSE, Swift and Fermi data.
- The high redshift GRBs (z > 2) can be used as a tool to determine the basic cosmological parameters, e.g. the Hubble law, in the future when the redshift defined sample volume will increase to a several thousands.
- For the long Swift GRBs  $T_{90}$  observed (without taking into account selection effects) durations distribution there is no the time dilation effect. The main selection effect is the uncertainty of spectral shape of the GRB at different redshifts and strong influence of the level of noise to determination of the  $T_{90}$ . Also incompleteness of GRBs sample which related to different nature of origin source (core collapse and binary merges) and can have different space, probabilistic and durations distributions.

Obviously we still have insufficient data in order to do final conclusions. The THESEUS project opens new perspectives in research of the Universe due to increase of redshift defined GRBs sample, which is shown in Figure 5.



Fig5. The estimated histograms of GRB number for 5 years due to THESEUS contribute from Amati L., et al. (2018) [1].

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