

# The new code of spectrum synthesis in magnetic stellar atmospheres with stratification of abundances

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**Abstract.** Here we present a new spectrum synthesis code *SynthM* for magnetic stars. *SynthM* works under the assumption of LTE in a plane-parallel atmosphere and is based on the non-magnetic spectrum synthesis code *SynthVa* of prof. V.Tsymbal. The *SynthM* code includes anomalous Zeeman pattern calculation, computation of local *IQUV* Stokes and disk integration algorithm. The transfer equation is solved by DELO method with quadratic approximation for the source function. The code allows use of two built-in analytical models of stellar magnetic field configuration (one of which identical to Piskunov's *SynthMag* model and the other is based on conception of "virtual" magnetic charges). Also, it is possible to use an arbitrary model of magnetic field configuration by setting the modulus and direction of field for each integrating node of a surface grid. Two integration methods are included in the code. *SynthM* allows one to calculate synthetic spectrum with preset stratification of abundances. *SynthM* is written in Fortran 95.

The comparison of calculated *IQUV* Stokes shows good agreement between the *SynthM* and *SynthMag* (Piskunov 1999) codes. The practical application of the designed code is illustrated by the example of synthetic spectrum test calculation for the region 6146–6150Å of the observed spectrum of  $\beta$ CrB.

**Key words:** stars: magnetic fields – stars: abundances – methods: analytical

## 1 Introduction

The magnetic field has a direct influence on formation of spectral lines in a stellar atmosphere. In accordance with the Zeeman effect, spectral lines are splitted into a number of components in different polarization:  $\pi$ -components and  $\sigma$ -components. Because through-passing radiation is polarized, the form of the transfer equation is changed. Besides, many magnetic stars have stratification of abundances in the stellar atmosphere. In the present paper we describe the *SynthM* code and some aspects of calculation taking into account the magnetic field and stratification of abundances.

## 2 Calculations

The code is based on the new version (*SynthVa*) of spectrum synthesis code *STARSP* (Tsymbal 1996). The Zeeman pattern and the relative strengths of components are calculated according to Sobelman (1977). It is important to mention that some of blue or red  $\sigma$ -components can be shifted in the opposite direction with respect to their prevalent shift direction. In the transfer equation part of the code we follow the techniques given by Rees et al. (1989). The Paschen-Back effect is not taken into account. The atomic transitions are taken from VALD database (Piskunov et al. 1995). The transfer equation is solved by DELO method with quadratic approximation for the source function which was suggested by Socas-Navarro et al. (2000) and

very well described by Piskunov & Kochukhov (2002). It is not necessary to cite the well-known formulae and equations in this paper. Note only that for approximation of the Voigt and Faraday-Voigt functions we use the fast Humlicek (1982) algorithm. The preset data on stratification of abundances are accounted for in calculation of ion concentrations and populations of atomic levels in atmosphere layers.

We use two disk integration algorithms in the reference frame shown in Fig. 1 ( $z$  points towards the observer, the  $y$  axis is in the plane which contains  $z$  and rotation axis). Both of them are based on the Gauss-Legendre quadrature technique. In the general case line profiles can be calculated as:

$$r(\lambda - \lambda_c) = \frac{\int_{-1}^{+1} \left\{ \int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} I(x, y, \lambda - \lambda_c - \lambda_c x (v_e/c) \sin i) dy \right\} dx}{\int_{-1}^{+1} \left\{ \int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} I_0(x, y) dy \right\} dx} . \quad (1)$$

By introducing a change of variables  $u = \frac{2y}{\sqrt{1-x^2}} - 1$ , it is possible to reduce the above formula to fixed limits of integration:

$$r(\lambda - \lambda_c) = \frac{\int_{-1}^{+1} \left\{ \int_{-3}^{+1} I(x, u, \lambda - \lambda_c - \lambda_c x (v_e/c) \sin i) (1 - x^2) du \right\} dx}{\int_{-1}^{+1} \left\{ \int_{-3}^{+1} I_0(x, u) (1 - x^2) du \right\} dx} . \quad (2)$$

For computation of similar integrals it is convenient to use the Gauss-Legendre quadrature. In case of uniform distribution of model parameters (abundances, magnetic field vector, etc.) over the stellar disk and  $v \sin i = 0$  km/s it is possible to use a simpler and faster calculated equation:

$$r(\lambda - \lambda_c) = \frac{\int_0^{+1} I(x, y, \lambda - \lambda_c) 2\pi x dx}{\int_0^{+1} I_0(x, y) 2\pi x dx} . \quad (3)$$

The code allows us to use both methods with any given number of nodes.

*SynthM* allows using two built-in analytical models of stellar magnetic field, one of which is identical to Piskunov's *SynthMag* model. This model is specified by three parameters of magnetic intensities  $B_r$ ,  $B_m$ ,  $B_l$  as:

$$\begin{aligned} B_x &= B_r \sin \theta + B_l \cos \theta \\ B_y &= B_m \\ B_z &= B_r \cos \theta - B_l \sin \theta . \end{aligned} \quad (4)$$

The other model is based on conception of virtual magnetic charges. This approach allows modeling centered magnetic dipole, shifted dipole, etc. The *SynthM* code uses equations given by Khalak et al. (2001). It is also possible to use arbitrary model of magnetic field by setting the modulus and direction of field for each integrating node of a surface grid.

### 3 Comparison

We have made comparison between the *SynthM* and the *SynthMag* codes because *SynthMag* was applied successfully to magnetic synthesis of spectra of CP stars. We used the model atmosphere of  $\gamma$ Equ with parameters:  $T_{eff} = 7700$  K,  $\lg g = 4.2$ ,  $\varepsilon(\text{Fe}) = -4.4$ ,  $\varepsilon(\text{Cr}) = -5.43$  and with the same magnetic field configuration as in *SynthMag*. Integration of intensities over the disk was carried out for 7 limb angles. We point out an important fact that calculations were conducted assuming the zero rotation velocity ( $v \sin i = 0$  km/s), zero microturbulence velocity and not convolved with the instrumental profile. All these factors cause smoothing of fine spectrum details (including Zeeman components) while any of these factors do not have direct relation to magnetic field. For example, using  $v \sin i$  about 2 km/s, we can find good agreement between spectra calculated with different magnetic field configuration. If the rotation velocity ( $v \sin i$ ) is about 10 km/s, then test calculation and comparison of codes from standpoint of validity in magnetic part have not any sense.

The first comparison was made for null magnetic field in order to determine any differences in damping calculations, disk integration algorithms, etc. The frame (a) in Fig. 2 shows the calculated profiles of Fe II  $\lambda 4923.927$ . The differences are quite small and maximum deviation of relative intensities ( $R = I/I_c$ ) is 0.004 in center of spectral line.

The second comparison was conducted for magnetic field configuration:  $B_r = 3000$ ,  $B_m = 2000$ ,  $B_l = 1000$  (G). The frame (b) in Fig. 2 shows disk-centre profiles, the frame (c) shows disk-integrated profiles. Maximum deviations in centre of Zeeman components are 0.002 and 0.0035, respectively. The comparison of disk-integrated Stokes  $QUV$  is shown in Fig. 3.

And finally, we have compared calculations of the spectral region 6147–6148Å which contains the spectral lines Cr II  $\lambda 6147.154$ , Fe II  $\lambda 6147.741$ , Fe I  $\lambda 6147.835$ . The field configuration is specified as:  $B_r = 5000$ ,  $B_m = 2000$ ,  $B_l = 0$  (G). The results are illustrated in Fig. 4. Maximum deviation is 0.006.

It seems that discrepancies of results are caused by different real numbers precision in the codes and different integration methods. Note that farther comparative tests using different model atmospheres and configuration of the magnetic field do not show any significant differences.

## 4 Application

We have calculated synthetic profiles of observed lines of the magnetic star  $\beta$ CrB for the region 6146–6150Å for the purpose of showing the practical application of the designed code.

We used the atmosphere model calculated in line-by-line way by the *LLModels* code (Tsymbal & Shulyak 2003) with the following parameters according to Savanov & Malanushenko (1990):  $T_{eff} = 8300$ K,  $\lg g = 4.0$ . *LLModels* makes it possible to compute a model atmosphere with individualized abundances. The abundances in atmosphere were specified according to Savanov & Malanushenko (1990) and Hardorp & Shore (1971). The lines list was extracted from VALD database (the strongest lines are Cr II  $\lambda 6147.154$ , Fe II  $\lambda 6147.741$ , Fe II  $\lambda 6149.258$ ). The value of  $v \sin i$  is  $3.5 \pm 1.5$  km/s (Bagnulo et al. 2000). We used the same configuration of the magnetic field as in *SynthMag*. The observed spectrum was obtained in CrAO (Crimea, Ukraine) at phase 0.087. The results are illustrated in Fig. 5.

The frame (a) in Fig. 5 shows comparison of the observed spectrum with the synthetic one computed using the microturbulence velocity  $v_{turb} = 0$  km/s, rotation velocity  $v \sin i = 5$  km/s, abundances  $\varepsilon(\text{Fe}) = -4.1$  and  $\varepsilon(\text{Cr}) = -4.4$ , magnetic field geometry:  $B_r = 5000$ ,  $B_m = 3000$ ,  $B_l = 0$  (G). The synthetic spectrum was convolved with gaussian (FWHM = 0.18) which corresponds to the comparison spectrum.

Comparison of the observed spectrum with the synthetic one calculated with the same parameters as mentioned above but for null magnetic field is shown in frame (b) in Fig. 5. It is clear from frames (a) and (b) that Zeeman splitting results in broadening of spectral lines (Zeeman broadening) and resolved split lines (if Zeeman splitting exceeds the rotational Doppler broadening). The observation of resolved Zeeman split spectral lines allows the mean magnetic field modulus to be detected.

Besides, the profile of a split line depends on the magnetic field geometry. Frame (c) in Fig. 5 shows comparison of the observed spectrum with the synthetic one calculated for magnetic field configuration:  $B_r = 6000$ ,  $B_m = 3000$ ,  $B_l = 0$  (G). One can see how profiles of spectral lines have changed in comparison with frame (a) in Fig. 5.

## 5 Conclusion

In present paper we have developed a code of spectrum synthesis in magnetic stellar atmospheres with stratification of abundances. The comparison of calculated  $IQUV$  Stokes shows a good agreement between the *SynthM* and *SynthMag* codes. The practical application of the designed code is illustrated by the example of synthetic profile test calculation of selected lines observed in the spectrum of  $\beta$ CrB.

The designed code, at first, was of independent importance as a tool for analysis of observed spectra, and secondly, will be contained as one of basic elements in calculation code of magnetic stellar atmospheres (which is now designed), as well as in a set of codes providing semiempirical construction of 3D models of stellar atmospheres.

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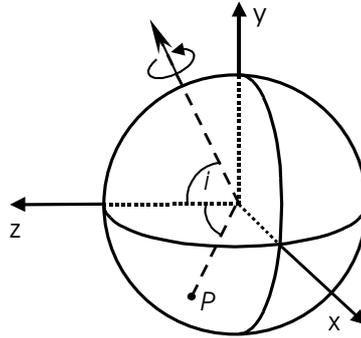


Figure 1: *Reference frame for disk integration algorithm.*

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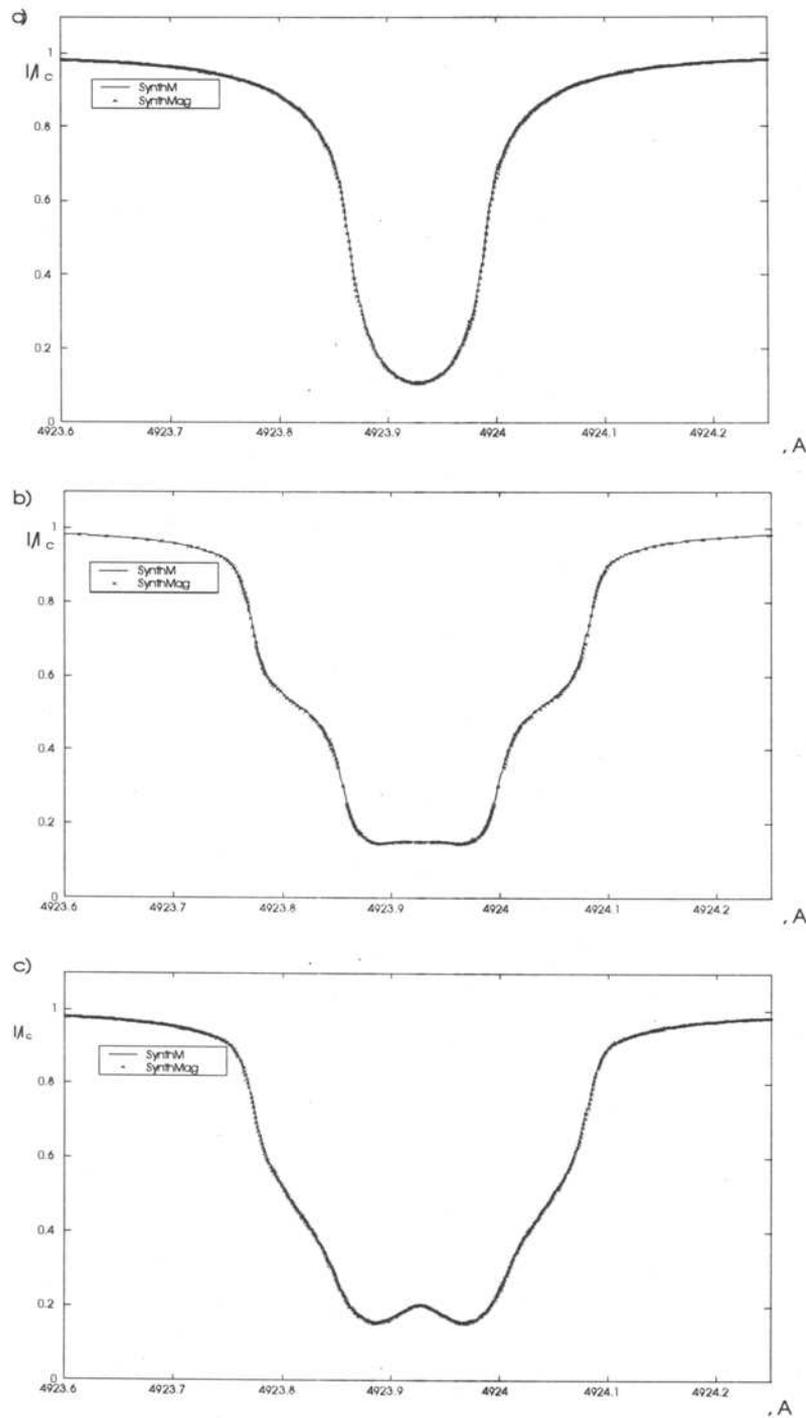


Figure 2: *Synthetic profiles of Fe II  $\lambda 4923$  calculated by SynthM and SynthMag. Frame (a) — comparison of disk-integrated profiles without magnetic field. Frames (b) and (c) — comparison of disk-centre profiles and disk-integrated profiles, respectively, for magnetic field configuration:  $B_r = 3000$ ,  $B_m = 2000$ ,  $B_l = 1000$  (G).*

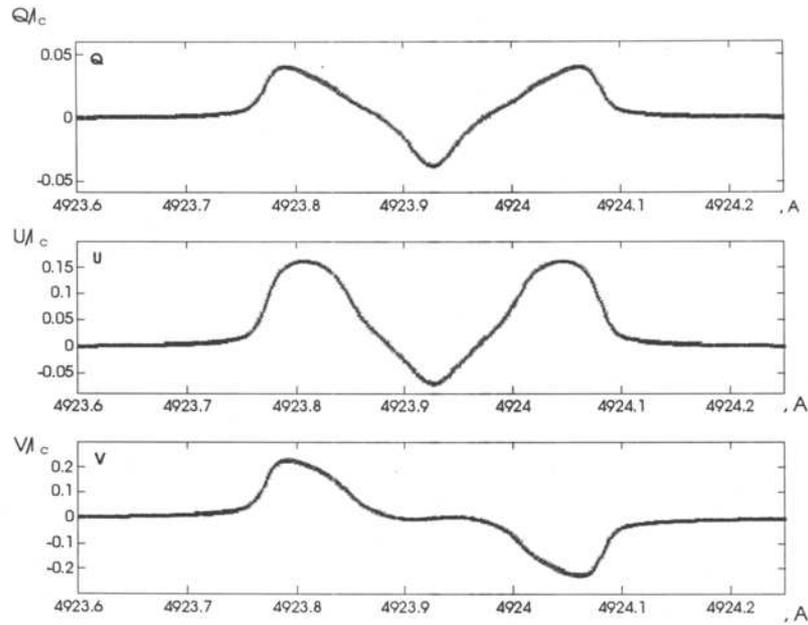


Figure 3: Comparison of disk-integrated Stokes QUV calculated by SynthM and SynthMag. Solid line — profiles calculated by SynthM, x-mark — profiles calculated by SynthMag. Magnetic field configuration:  $B_r = 3000$ ,  $B_m = 2000$ ,  $B_l = 1000$  (G).

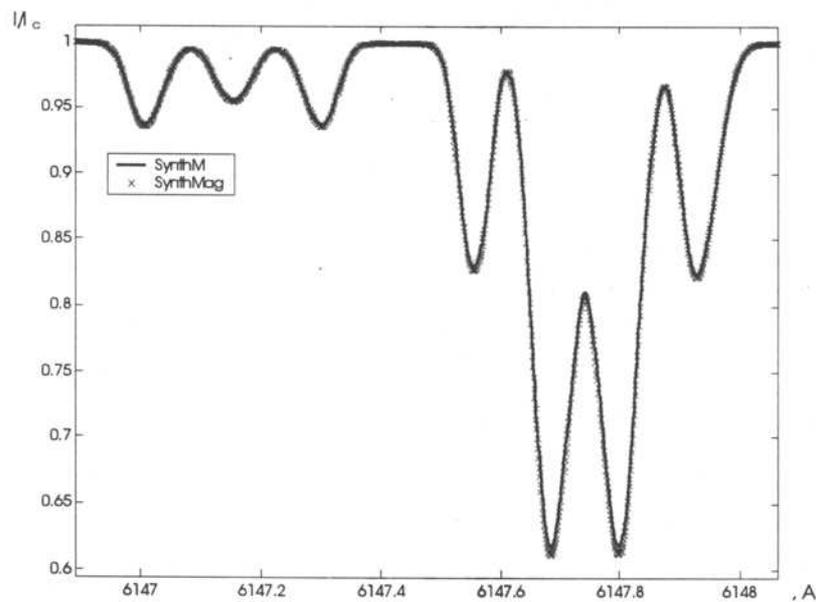


Figure 4: Comparison of synthetic spectra calculated by SynthM and SynthMag for the spectral region  $\lambda 6147-6148$ , magnetic field configuration:  $B_r = 5000$ ,  $B_m = 2000$ ,  $B_l = 0$  (G).

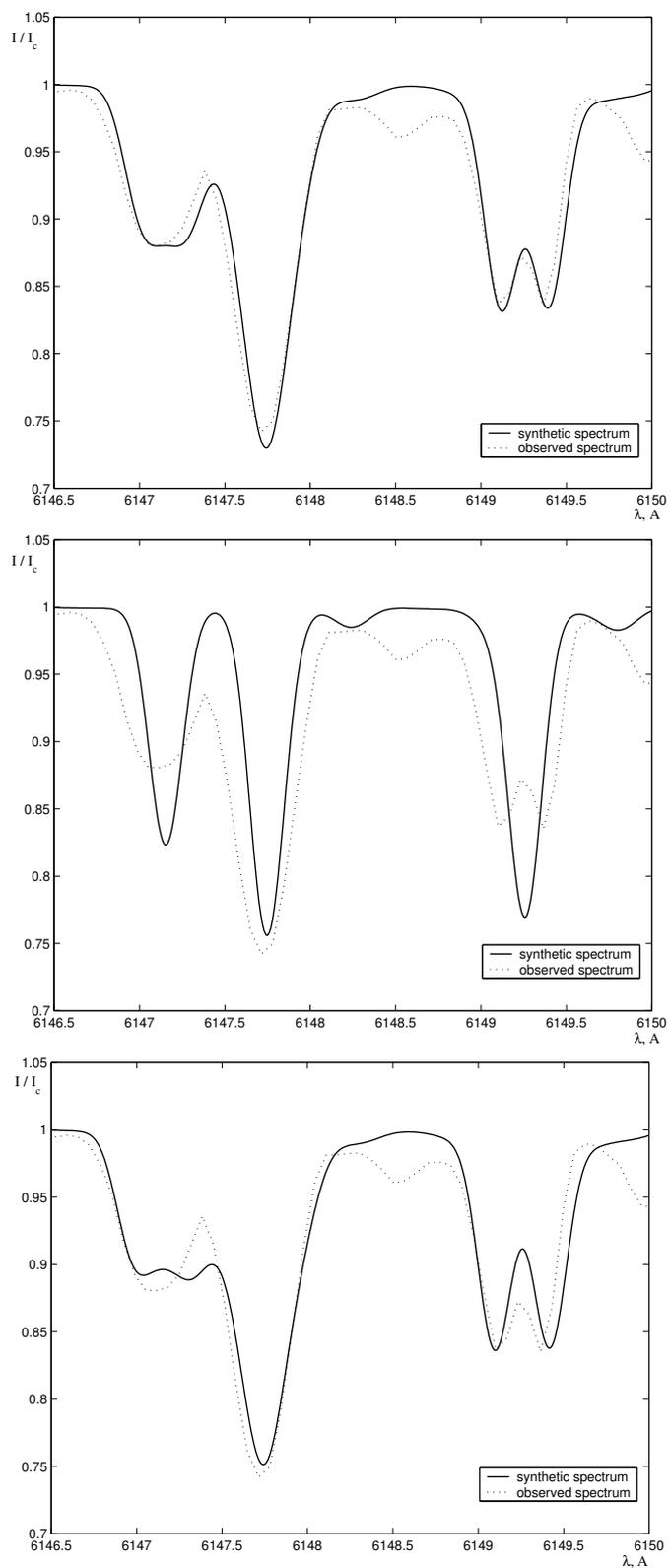


Figure 5: Comparison of synthetic and observed spectra of  $\beta\text{CrB}$ . Frame (a) — magnetic field configuration:  $B_r = 5000$ ,  $B_m = 3000$ ,  $B_l = 0$  (G). Frame (b) — null magnetic field. Frame (c) — magnetic field configuration:  $B_r = 6000$ ,  $B_m = 3000$ ,  $B_l = 0$  (G).